Definition

**Scalars and vectors:** Scalar is a single real number called magnitude and is not related to any direction in space. The vector is a quantity which has a magnitude as well as a definite direction in space. The speed of a bus is a scalar quantity but the velocity is a vector quantity.

**Line vectors:** The vector $\overrightarrow{AB}$, A is called the origin and B the terminus. The magnitude of the vector is given by the length AB and its direction is from A to B. These vectors are called line vectors.

**Equal vectors:** Two vectors are said to be equal when they have the] same length (magnitude) and are parallel having the same sense of direction. The equality of two vectors is written as $\vec{a} = \vec{b}$.

**Zero vectors:** If the origin and terminal points of a vector are same, then it is said to be a zero vector. Evidently its length is zero and its direction is indeterminate.

**Unit vector:** A vector is said to be a unit vector if its magnitude be of unit length.

**Position vector:** The position vector of any point P, with reference to an origin O is the vector $\overrightarrow{OP}$. Thus taking O as origin we can find the position vector of every point in space. Conversely, corresponding to any given vector $\vec{r}$ there is a point P such that $\overrightarrow{OP} = \vec{r}$.

**Addition of two vectors:** Let $\vec{a}$ and $\vec{b}$ be two vectors with respect to the origin O. The sum of these two vectors is given by $\vec{a} + \vec{b}$. 
The unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$: The vectors $\vec{i}$, $\vec{j}$, $\vec{k}$ have unit magnitude and they lie on the x, y and z axes respectively. We can expresses any vector in terms of these three unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$.

Collinear vectors: Two vectors $\vec{a}$ and $\vec{b}$ are said to be collinear if $\vec{a} = \lambda \vec{b}$, for some scalar $\lambda$, i.e., two vectors are collinear if the coefficient of $\vec{i}$, $\vec{j}$ and $\vec{k}$ are proportional.

Magnitude of a vector: Let $\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$. Then the magnitude or length of the vector $\vec{a}$ is denoted by $|\vec{a}|$ or $a$ and is defined as $a = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Distance between two points: Let $P_1$ and $P_2$ be two points whose position vectors are respectively $\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$ and $\vec{b} = b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k}$. Then the vector $\vec{P_1P_2} = \vec{b} - \vec{a}$ is the position vector of $P_2$ - position vector of $P_1 = (b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k}) - (a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k})$.

Then the distance between two points $P_1$ and $P_2$ is the magnitude of the vector $\vec{P_1P_2}$.

*** The unit vectors in the direction of a vector $\vec{a}$ are given as $\pm \frac{\vec{a}}{|\vec{a}|}$.

Examples:

1. Find the value of $q$ if $\vec{a} = 2\vec{i} + 5\vec{j} + q\vec{k}$. If the magnitude of $\vec{a}$ is 9.

2. Given vectors $\vec{a} = 5\vec{i} + \vec{j} + 3\vec{k}$, $\vec{b} = \vec{i} - 3\vec{j} + 4\vec{k}$ and $\vec{c} = 7\vec{i} + 2\vec{j} - 3\vec{k}$. Find the unit vector in the direction of $\vec{a} - \vec{b} + 2\vec{c}$.

3. Find the distance between A and B whose-position vectors are $\vec{a} = 5\vec{i} + \vec{j} + 3\vec{k}$, $\vec{b} = \vec{i} - 3\vec{j} + 4\vec{k}$ respectively.

4. Prove by vector method that the three points $A$ (2, 3, 4), $B$ (1, 2, 3) and $C$ (4, 2, 3) form a right-angled triangle.
5. Show that the three points $-3i - 6j + 21k$, $9i + 3k$ and $15i + 3j - 6k$ are collinear.

**Scalar Product or Dot Product**

The scalar or dot product between vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a} \cdot \vec{b}$ and defined as

$$\vec{a} \cdot \vec{b} = ||\vec{a}|| ||\vec{b}|| \cos \theta,$$

where $\theta$ is the angle between $\vec{a}$ and $\vec{b}$. The value of $\vec{a} \cdot \vec{b}$ is a scalar quantity.

*** Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular if and only if $\vec{a} \cdot \vec{b} = 0$

**Examples:**

1. Given vectors $\vec{a} = 5i + j + 3k$, $\vec{b} = i - 3j + 4k$ and $\vec{c} = 7i + 2j - 3k$.

   Find (i) $\vec{a} \cdot \vec{b}$ (ii) $(\vec{a} \cdot \vec{b}) \cdot \vec{c}$ (iii) angle between $\vec{a} - \vec{c} + 2\vec{b}$ and $\vec{a} + \vec{b} + \vec{c}$

2. Find the value of $q$ for which the two vectors are $\vec{a} = 5i + qj + 3k$ and $\vec{b} = i - 3j + 4k$ are perpendicular to each other.

**Vector Product or Cross Product**

The vector product or cross product between two vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a} \times \vec{b}$ and is defined by

$$\vec{a} \times \vec{b} = ||\vec{a}|| ||\vec{b}|| \sin \theta,$$

where $\theta$ is the angle between $\vec{a}$ and $\vec{b}$.

**NOTE:**

(i) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

(ii) If $\vec{a} \times \vec{b} = 0$, then $\vec{a}$ and $\vec{b}$ are parallel or collinear

(iii) $\vec{a} \times \vec{a} = 0$

(iv) If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a parallelogram then its area is $|\vec{a} \times \vec{b}|$.

(v) If $\vec{a}$ and $\vec{b}$ represent any two sides of a triangle then its area is $\frac{1}{2}|\vec{a} \times \vec{b}|$. 
(vi) If \( \vec{a} \times \vec{b} = \vec{c} \) then \( \vec{c} \) is perpendicular to both \( \vec{a} \) and \( \vec{b} \).

(vii) \( \vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} \)

(viii) Three vectors \( \vec{a} \), \( \vec{b} \) and \( \vec{c} \) are said to be coplanar, if \( \vec{a} \cdot (\vec{b} \times \vec{c}) = 0 \).

**Examples**

1. If \( \vec{a} = 5i + j + 3k \) and \( \vec{b} = i - 3j + 4k \). Find \( \vec{a} \times \vec{b} \).

2. If \( \vec{a} \) and \( \vec{b} \) are two vectors such that \( |\vec{a}| = 16 \), \( |\vec{b}| = 12 \) and \( \vec{a} \cdot \vec{b} = 0 \). Find \( |\vec{a} \times \vec{b}| \).

3. Find the area of the triangle two of whose sides are given by the vectors \( \vec{a} = 5i + j + 3k \) and \( \vec{b} = i - 3j + 4k \).

4. Find the area of the parallelogram formed by two vectors \( \vec{a} = 5i + j + 3k \) and \( \vec{b} = i - 3j + 4k \).

5. Find the unit vector perpendicular to each of the vectors \( \vec{a} = 5i + j + 3k \) and \( \vec{b} = i - 3j + 4k \).

6. Find a vector of magnitude 9 perpendicular to both the vectors \( \vec{a} = 5i + j + 3k \) and \( \vec{b} = i - 3j + 4k \).

7. Show that the vectors \( \vec{a} = 4i + 2j + k \), \( \vec{b} = 2i - j + 3k \) and \( \vec{c} = 8i + 7k \) are coplanar.

8. If a force given by \( \vec{F} = 5i + j + 3k \) displaces a particle from the position B to C whose position vectors are \( \vec{b} = i - 3j + 4k \) and \( \vec{c} = 7i + 2j - 3k \) respectively. Find the work done by the force.

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