COURSE CODE: **PHS 211**

COURSE TITTLE: **CLASSICAL PHYSICS**

NUMBER OF UNITS: **2 Units**

COURSE DURATION: **Two hours per week**

COURSE DETAILS:

Course Coordinator: **Dr. O.I. Olusola B.Sc., M.Sc. Ph.D.**

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Office Location: **Room C210, COLNAS**

Other Lecturers: **Nill**

COURSE CONTENT:

An introduction to classical mechanics, space and time, straight line kinematics, motion in a plane; forces and conservation particle dynamics, collisions and conservation laws, work and potential energy, inertia forces and non-inertia frames; central focus motions, rigid bodies and rotational dynamics.

COURSE REQUIEREMENTS:

This course is compulsory for all students pursuing B.Sc. Degree in Physics in the University. In this light, students are expected to participate in all the course activities and obtain minimum of 75% attendance to be able to write the final examination.

READING LIST:

1. Golstein G,Classical mechanics, McGrawhill, 1983
2. Plumpton, C., Tomkys, W.A., Theoretical Mechanics for sixth forms, 2nd Ed., Pergamon Press, New York
3. Harper, C., Introduction to Mathematical Physics, Prentice-Hall of India, New Delhi, 1989.
4. Chorton, F.A textbook of dynamics, Ellis Horwood Ltd. New York, 1988

LECTURE NOTES

1.0 INTRODUCTION TO CLASSICAL MECHANICS

Mechanics is the study of the effects of external forces on bodies at rest or in motion. Such bodies could be rigid or elastic (solids), liquids or gases. The quantitative concepts used in mechanics can be classified into two groups:

(1) The FUNDAMENTAL CONCEPTS – consisting of length, mass & time and
(2) The DERIVED CONCEPTS which include other terms such as density, area and speed.

By understanding classical mechanics, a student prepares himself to pursue the fields of space physics, relativistic mechanics, statistical mechanics, acoustics, elasticity and fluid mechanics all of which can be traced to Newton, laws of motion.

Mechanics play a far wider role in physical sciences. Classical mechanics is other wisely termed Newton in mechanics.

THE CLASSICAL MODELS OF TIME & SPACE

In the Newtonian (classical) models, intervals of time are modeled by real numbers and these real numbers are measured by instruments known as clocks. The real number which models a given interval of time will depend on the unit of time used to calibrate the clocks. The SI unit of time is second.

SCALAR & VECTOR

Quantities with magnitudes only are called scalars while these with both magnitude and direction are referred to as vectors.

Notation of vectors

In 1 – dim. x = x**x**

Where x = 1**x**1 and x is a unit vector in x – direction i.e. along x – axis.

In 3 – dim r = (x, y, z) in Cartesian coordinate

(r, θ, □) in spherical polar coordinate

(r, θ, z)

V = λv' where λ is a scalar quantity

In parallel, then v = λ'V', λ = scalar

ADDITION OF VECTORS

If r_1 and r_2 are two vectors, then

$r = r_1 + r_2 = r_2 + r_1$ cummutative law is obeyed under addition of vectors

If r_1 and r_2 are inclined at angle β, then cosine rule can be applied to determine the resultant vector **r** as

$$\mathbf{r} = r^2{}_1 + r^2{}_2 - 2r_1\, r_2 \cos\beta$$

COMPONENT OF A VECTOR

$F = (F_x, F_y)$ in cartezian coordinate syst.

$F_x = F\cos\theta$

$F_y = F\sin\theta$

In plane polars

$F = (F, \theta)$

In 3 – dim

$F = (F_x, F_y, F_2)$ in cartezian coordinate system

$F = (F, \theta, Л)$ in spherical polars

Where

$F_x = F\sin\theta\cos$ □

$F_y = F\sin\theta\sin\theta$

$F_z = F\cos\theta$

di = volume element

= dx dy dz in ordinary space of the Cartesian coordinate system

= $r^2\sin\theta drd\theta d$□ i.e. volume elememt in ordinary space in spherical polars.

VECTOR NOTATIONS

i= unit vector along x-direction

= **v**x

j = a unit vector along y-direction

= **v**y

V = **v**cos θi – **v** sin θ j

$V = (V_x, V_y, V_z) = (V_{xi} + V_{yj} + V_{zk})$

Direction cosine

If the vector **v** makes an angle λ with x-axis, β with y-axis & and angle γ z-axis, then

$V_x/v = \cos \lambda$

$V_y/v = \cos \beta$

$V_z/v = \cos \gamma$

Since $\cos^2 \lambda + \cos^2\beta + \cos^2 \gamma = 1$, then the angle which various components make the respective direction is called the direction cosine.

ADDITION OF SEVERAL VECTORS

$r = r_1 + r_2 + r_3 + \ldots\ldots\ldots\ldots\ldots\ldots.$

$= (r_1x + r_2x + r_3x + \ldots..)\, i + (r_1y + r_2y + r_3y + \ldots..)j+$

SCALAR OR DOT PRODUCT

If there are two vectors **m** and **n**, then **m**. **n** = mn cos θ = **n**. **m** i.e. dot product is cummutative

Where m = |m| and n = |n|

Properties of dot product

(1.) If k is another vector, the dot product is distributive with respect to. **k** i.e.

k.(**m** + **n**) = **k**. **m** + **k**. **n**

(2.) Dot product is cummutative

m. **n** = **n**. **m**

VECTOR PRODUCT

Vector product of **m** and **n** is defined as

m X **n** = (mn sinθ)k

If $m = (m_x, m_y, m_z)$

$n = (n_x, n_y, n_z)$

Then, $m \times n = (m_y\, n_z - m_z\, n_y)\, I - (m_x\, n_z - m_z\, n_x)j + (m_xn_y - m_y\, n_x)k$

Properties

(1) In vector product: order of multiplicate is important i.e. **m** X **n** + **n** X **m** hence it is non-cummutative

(2) It is distributive i.e. **k** X (**m** + **n**) = **k** X **m** + **k** X **n**

CHAPTER TWO

2.0 KINEMATICS

Mechanics deals with relations of force, matter and motion. The branch mechanics that describe motion without reference to force is "Kinematics". Rest and motion are relative,, therefore, we need a frame of reference.

Rectilinear motion i.e motion in a straight line

Average velocity between points A and B is denoted by V, which is defined

$$U = \frac{x_1 - x_0}{t_1 - t_0} = \frac{\Delta x}{\Delta t} \ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

Instantaneous velocity i.e velocity at a particular time

$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

In general, V depends on time i.e V is a function of time i.e

$V = V(t)$

Then:

$dx = V(t)\,dt$

$$\int_{x0}^{x1} dx = \int_{t0}^{t1} v(t)\,dt$$

$$[x(t)]_{x0}^{x1} = \int_{t0}^{t1} v(t)\,dt$$

$$\Rightarrow x(t) - x(t_0) = \int_{t0}^{t1} V(t)\,dt$$

x is measured in metres, t in sees and V in ms^{-1}.

Example: Suppose $x(t) = (at^2 + C)\ \hat{x}$ where a & c are constants.

$$V(t) = \frac{dx}{dt} = (2at)\ \hat{x}$$

$$\bar{V} = \frac{\Delta x}{\Delta t}$$

$$= \frac{x(t) - x(t_0)}{t - t_0}\ \hat{x}$$

$$= \frac{a(t^2 - t_0^2)}{t - t_0}\ \hat{x}$$

$$= \frac{a(t - t_0)(t + t_0)}{t - t_0}\ \hat{x}$$

$$= a(t + t_0)\ \hat{x}$$

Acceleration: Uniform acceleration $\Rightarrow$ $V \neq V(t)$

In general, however, V depends on time i.e. $V = V(t)$

$$\bar{a} = \frac{V(t) - V(t_0)}{t - t_0} = \frac{\Delta v}{\Delta t} \quad \ldots\ldots\ldots\ldots\ldots\ (3)$$

$$a = a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

If $a \neq a(t)$, then it is uniform acceleration. If V increases as t increases, then we have acceleration but if V decreases as t increases, then it is deceleration or retardation.

$$a(t) = \frac{dv(t)}{dt}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; (5)$$

$$\Rightarrow \int_{v0}^{v} dv(t) = \int_{t0}^{t} a(t)dt$$

i.e. $V(t) - V(t_0) = \int_{t0}^{t} a(t)dt$

Recall from (4) and (2) that

$dv(t) = a(t)dt$ ……………..(a)

$v(t) = {}^{dx(t)}/_{dt}$………………(b)

$\Rightarrow v(t)\, dv(t) = a(t)\, dt \cdot {}^{dx(t)}/_{dt}$ ………………(6)

$\Rightarrow V \cdot dv = a \cdot dx$

$$\int_{v0}^{v} V dv = \int a \cdot dx$$

$$\Rightarrow \frac{1}{2}[\, v2]^{v}_{vo} = \int a \cdot dx$$

Hence, knowing a (x) r. h. s can be calculated, the acceleration, a, is measured in ms^{-2}.

2.1 CURVILINEAR MOTION

Let us consider a particle describing a curved path P as shown in the fig. below, At time t, the particle is at A

At time t^1, the particle is at B

Clearly, $\underline{r} = xi + yj + zk$

Similarly, $\underline{r}^1 = xi + yj + zk$

Displacement of particle = $A\underline{B} = \Delta r$

$AB = \Delta\underline{r} = \underline{r}' - \underline{r}$

$= (x' - x)I + (y^1 - y)\underline{j} + (z^1 - z)\underline{k}$

$= \Delta xi + \Delta yj + \Delta zk$

Average velocity, $\underline{V} = \frac{\Delta r}{\Delta t} = (\frac{\Delta k}{\Delta t})\underline{i} + (\frac{\Delta y}{\Delta t})\underline{j} + (\frac{\Delta z}{\Delta t})k$ ___________ (7)

Direction of $\underline{v}$ is parallel to Δr

Instantaneous velocity = $\underline{v} = \lim \Delta tmo \frac{\Delta v}{\Delta t}$ __________ (8a)

As $\Delta t \longrightarrow 0$; point $B \longrightarrow A$

$\underline{AB} = \Delta r$ changes continuously in magnitudes and direction and so does $\underline{v}$

In the limet when B is close to A, Δr coincides in direction with tangent AT. Thus, in curvilinear motion, the instantaneous velocity is a vector tangent to the path and is given by

$\underline{V} = \frac{dv}{dt}$

$= (\frac{dx}{dt})\underline{i} + (\frac{dy}{dt})j + (\frac{dz}{dt})k$ ______________(8b)

$V = |\underline{V}| = (V_x^2 + V_y^2 + V_z^2)^{1/2}$ ________________(9)

Where $V_x = \frac{dx}{dt}$, $V_y = \frac{dy}{dt}$ and $V_z = \frac{dz}{dt}$

Alternatively, from 8(a): $\underline{V}$ = lim (delta t to 0) $\frac{\Delta r}{\Delta t}$

= lim (delta t $\longrightarrow$ 0) $(\frac{\Delta r}{\Delta s} . \frac{\Delta s}{\Delta t})$

= (lim dt $\longrightarrow$ 0 $\frac{\Delta r}{\Delta s}$) (lim dt $\longrightarrow$ 0 $\frac{\Delta s}{\Delta t}$)

$= \frac{ds}{dt} U_T$ _______________________(10)

= V u_T

Speed direction

where U_T is a unit rector along the tangent.

2.2 **Average Acceleration**

We define average acceleration as

$\underline{a} = \frac{\Delta v}{\Delta t} = (\frac{\Delta vx}{\Delta t})i + (\frac{\Delta vy}{\Delta t})j + (\frac{\Delta vz}{\Delta t})k$

= axi + ayj + azk ______________ (11)

$\underline{a} = \frac{V - V}{t1 - t} = \frac{\Delta v}{\Delta t}$

The direction of the acceleration is always in the direction of concavity of the curve.

This, $\underline{a}$ is parallel to $\Delta\underline{V}$

Instantaneous acceleration = $\underline{a}$ = lim Δt-0 $\frac{\Delta v}{\Delta t}$

$= (\frac{dvx}{dt})I + (\frac{dvy}{dt})j + (\frac{dvz}{dt})k = \frac{dv}{dt}$ __________________(12)

= axi + ayj + azk

$a = [a_x^2 + a_y^2 + a_z^2]^{1/2}$

Where

$a_x = {dvx}/{dt}$, $a_y = {dvy}/{dt}$, $a_z = {dvz}/{dt}$

Also

$\underline{a} = \frac{dv}{dt} = \frac{d}{dt} . (\frac{dx}{dt}) = \frac{d^2y}{dt^2}$

2.3 MOTION UNDER CONSTANT ACCEL ERATION

If a constant acceleration is constant, then from

$\underline{a} = {dv}/{dt}$ ---1

$d\underline{v} = \underline{a}.dt$

$\underline{r}' - \underline{r}$ = displacement

$\int_{v0}^{v} dv = \int_{t0}^{v} a\,dt$

i.e $(\underline{V}) = \underline{a} \int_{t0}^{v} dt$

from Eq. 1 above

$dv(t) = \underline{a}(t)\ dt$……………….(a)

$v(t) = {dx(t)}/{dt}$ …………...(b)

$v(t)\ dv(t) = \underline{a}(t)\ dt. \frac{dx(t)}{dt}$ ________________________________(2)

$\underline{v}.dv = \underline{a}.\ dx$

$\int_{v0}^{v} v.dv = \int a.dx$

$\left(\frac{1}{2} V^2\right)_{V0}^{V} = \int a.dx$

Hence, knowing $\underline{a}(x)$, r.h.s can be calculated. a measured in ms^{-2}.

Acceleration: uniform acceleration $V \neq V(t)$.

In general, however, V depends on time i.e $V=V(t)$

$$a = \frac{\Delta v}{\Delta t} \text{ -----------------------(3)}$$

$$A = a(t) \lim_{\Delta t \to 0} a\left(\frac{\Delta v}{\Delta t}\right) = \frac{dv}{dt} \text{ -------------------------------------(4)}$$

If $a \neq a(t) = \frac{dv(t)}{dt}$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \text{ --(5)}$$

$$\int_{v0}^{v} dv(t) = \int_{t0}^{t} a(t)dt$$

i.e $Vt - v(t_0) = \int_{t0}^{t} a(t)dt$

Recall from Eqs. (4) and (2) that

$\underline{V} - \underline{V}_o = a(t - t_0)$

i.e $\underline{V} = \underline{V}_o + a(t - t_0)$

also,

$\underline{V} = {}^{dr}/_{dt}$, we have

$$\int_{r0}^{r} dr = \int_{t0}^{t} V dt$$

$$\underline{r} - r_0 = \int_{t0}^{t} \left[V_o + a(t - t_o)\ dt\right]$$

$$= V_0(t - t_0) + [\tfrac{1}{2} at^3]^t \_\_\_\_ at_0t/^t$$

$$= V_0(t - t_0) + \tfrac{1}{2}a[t^2 - t_0^2] - at_0(t - t_0)$$

$$= V_0(t - t_0) + \tfrac{1}{2}a[t^2 - t_0^2] - at_0t + at_0^2$$

$$= V_0(t - t_0) + \tfrac{1}{2}at^2 - \tfrac{1}{2}at_0^2 - at_0t + at_0^2$$

$= V_0 (t - t_0) + \frac{1}{2}at^2 - at_0t + \frac{1}{2}at_0^2$

$= V_0 (t - t_0) + \frac{1}{2}a (t - t_0)^2$

$\Longrightarrow \underline{r} = \underline{r}_0 + v_0 (t - t_0) + \frac{1}{2}a (t - t_0)^2$ ______________________ **(6)**

WORKED EXAMPLES:

2.4 MOTION OF A PROJECTILE

Let us consider a projectile describing a path, p as shown in the figure below:

$V_0 = V_0x_i + V_0yj = (V_0 \cos \propto)i + (V_0 \sin \propto)_j$ --(1)

At time, $t_0 = 0$

$V = Vxi + Vyj$

$= (V_0xi + V_0yj) - gtj$

$= V_0xi + (V_0y - gt)j$

$$\left.\begin{array}{l} \Longrightarrow V_x = V_0x \ldots\ldots(a) \\ \\ \text{And } V_y = V_0y - gt \ldots.(b) \end{array}\right\} \text{\_\_\_\_\_\_\_\_\_\_\_\_ (2)}$$

In a similar manner, with $r_0 = 0$ and $t_0 = 0$, we have $r = V_0t + \frac{1}{2}at^2$

i.e. $r = (V_0xi + V_0yj)t - \frac{1}{2}gt^2$

Hence,

$$\left.\begin{array}{l} X = V_0xt \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (a) \\ \text{And } y = V_0yt \_\_\_\_ \frac{1}{2}gt^2 \ldots\ldots (b) \end{array}\right\} \text{\_\_\_\_\_\_\_\_\_\_ (3)}$$

2.4.1 TIME REQUIRED TO REACH THE PEAK

To reach the time required to reach the highest point A, we will need to set $V_y = 0$ in (16a)

i.e. $V_0y - gt = 0$

$t = V_0y/g$

Recall $V_0y = V_0 \sin \propto$

$$t = \frac{V_0 \sin\propto}{g} \quad \text{------------------------ (4)}$$

2.4.2 MAXIMUM HEIGHT, H

The maximum height is obtained by substituting the value for t in (3b) and we have

$$H = V_0y. \frac{V_0 \sin\propto}{g} - \frac{1}{2}g\, V_0^2 \frac{\sin^2\propto}{g^2}$$

$$H = V_0^2 \frac{\sin^2\propto}{g} - V_0^2 \frac{\sin^2\propto}{2g}$$

i.e.

$$H = V_0^2 \frac{\sin^2\propto}{2g} \quad \text{--------------------------------(5)}$$

2.4.3 TIME OF FLIGHT

The time of required for the particle to return to the ground level at B is called time of flight. This can be obtained by setting y=0 in (17b) and we obtained

$$0 = V_0yt - \frac{1}{2}gt^2$$

$$\Longrightarrow \frac{1}{2}gt^2 = V_0yt$$

$$\Longrightarrow t_{total} = \frac{2V_0y}{g}$$

$$\frac{2V_0\sin^2\propto}{g} \text{ --- } (6)$$

2.4.4 RANGE, R

The range R=OB is obtained by substituting t_{total} from (6) into (17a) and get

$$R = V_0x \times \frac{2V_0\sin\propto}{g}$$

$$= \frac{2V_0^2\sin\propto\cos\propto}{g} \quad [\text{since } V_0x = \underline{\cos}\propto]\text{------------------------------ } (21)$$

R is maximum when $\propto = 45^0$ or $\frac{\pi}{4}$