CVE 303 Hydraulics I (3 Units)

- Fundamental principles of hydraulics: continuity, energy and momentum.
- Open channel flows: uniform flow, gradually varied flow.
- Flow resistance. Mannings and Chezy's equations.
- Application of the energy, momentum, and continuity equations in combination.
- Specific energy.
- Flow in conduits; classification of laminar and turbulent flows.
- Losses at inlets, bends, outlets etc.
- Application of continuity, energy and momentum to closed conduit flow.
- Pipe systems; reservoir/pipe combinations.
- Hardy Cross flow measurements.

Lecturer: Dr. O.S. Awokola

Lectures:

Tutorial: *To be decided*

Assignments:

Gradings:	Assignments	0-5%
	Midterm	25% (likely to be 2 tests or one plus snap tests)
	Final	70%

References:

(1) Fluid Mechanics: J.F. Douglas, J.M. Gasiorek & J.A. Swaffield

(2) Fluid Mechanics, Victor L. Streeter, E. Benjamin

(3) Fluid Mechanics With Engineering Applications, Robert L. Daugherty & Joseph B. Franzini

(4) Schaum's Outline Series Fluid Mechanics and Hydraulics (SI(metric) edition Renald V. Giles

CHAPTER ONE: BRIEF REVISION

Fluid mechanics may be divided into **three** branches:

1. Fluid Static is the study of the mechanics of fluids at rest
2. Kinematics deals with velocities and streamlines without considering forces or energy
3. Hydrodynamics is concerned with the relations between velocities and accelerations and forces exerted by or upon fluids in motion.

FLUID FLOW

The motion of a fluid is usually extremely complex.

(i) **Uniform flow:** If the velocity at a given instant is the same in magnitude and direction at every point in the fluid.

(ii) If at the given instant the velocity changes from point to point the flow is described as **Non-Uniform.**

(iii) Steady flow is one in which the velocity, pressure and cross-section of the stream may vary from point to point **but do not change with time.**

(iv) If at a given point conditions do change with time the flow is described as **unsteady.**

There are therefore 4 possible types of flow.

(a) ***Steady Uniform Flow***: Conditions do not change with position and time

(b) ***Steady Non-uniform flow****:* Conditions change from point to point but not with time

(c) **Unsteady Uniform:** At a given instant of time the velocity at every point is the same, but this velocity will change with time.

(d) **Unsteady-Non-uniform flow**: The cross-sectional area and velocity vary from point to point and also change with time.

MOTION OF A FLUID PARTICLE

Newton's laws : When a force is applied any particle or element of fluid will obey the normal laws of mechanics in the same way as solid body.

(i) A body will remain at rest or in a state of uniform motion in straight line until acted upon by an external force.

(ii) The rate of change of momentum of a body is proportional to the force applied and takes place in the direction of action of that force.

(iii) Action and reaction are equal and opposite.

*Momentum is the product of mass and velocity

Force=mass x acceleration

$$F = ma$$

LAMINAR AND TURBULENT FLOW

Observation shows that two entirely different types of flow exist. This was demonstrated by Osborne Reynolds in 1883.

When $R_e \leq 2000$ Flow is Laminar, but when $R_e \geq 2000$ the flow is Turbulent in pipes.

CONTINUITY OF FLOW

Except in nuclear processes, matter is neither created nor destroyed. This principle of conservation of mass can be applied to a flowing fluid.

CONTINUITY EQUATION $a_1 v_1 = a_2 v_2$

BERNOULLI'S EQUATION states that for steady flow of a frictionless fluid along a streamline, the total energy per unit weight remains constant.

P= Pressure

V=Velocity

mg= Weight

Potential energy due to height= zmg

Kinetic energy due to velocity $= \frac{1}{2}\left(\frac{mg}{g}\right)v^2 = \frac{mv^2}{2} = \frac{1}{2}mv^2$

Divide through by weight (mg)

Potential energy per unit weight=z

Kinetic energy per unit weight= $\frac{v^2}{2g}$

Pressure energy per unit weight = $\frac{P}{\rho g}$

H=Constant=Total energy per unit weight

$H = \frac{P}{\rho g} + \frac{V^2}{2g} + z$ (m)=Pressure head plus Velocity head plus Potential head

MOODY'S CHART

Friction factor f'

Reynolds Number Re

Relative Roughness= $\frac{\varepsilon}{D}$

Laminar Zone

Transition zone

Complete Turbulent zone

Darcy Weisbach Equation for head loss due to friction

$$h_f = f'\frac{LV^2}{d2g}$$

$$f' = \phi(\mathrm{Re})$$

Laminar flow $f' = \frac{64}{\mathrm{Re}}$

FRICTION FACTOR

Referring to Moody's Chart for friction factor for pipe, the chart shows that there are four zones.

1. Laminar flow where $f' = \frac{64}{\text{Re}}$ n
2. A critical range where the values are uncertain because the flow might be either laminar or turbulent
3. Transition zone, where f' is a function of both Reynolds number and Relative pipe roughness
4. A zone of complete turbulence where the value of f' is independent of Reynolds Number and depends SOLELY upon Relative Roughness

There is no sharp line of demarcation between the transition zone and the zone of complete turbulence.

(a) For smooth pipes Blasius suggests for Re between 3,000 and 10,000 $f' = \frac{0.316}{\text{Re}^{1/4}}$

(b) For values of Re up to 3.000,000 von Karman's equation modified by Prandtl is $\frac{1}{\sqrt{f'}} = 2\log\left(\text{Re}\sqrt{f'}\right) - 0.8$

(c) For all pipes the Hydraulic Institute and many engineers consider the Colebrook equation reliable when evaluating f' $\frac{1}{\sqrt{f'}} = -2\log\left[\frac{\varepsilon / D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f'}}\right]$

It can be observed from Colebrook equation that :

(i) For smooth pipes where the value of $\frac{\varepsilon}{D}$ is very small, the first term in the bracket can be neglected.

(ii) If Re is very large the second term in the bracket can be neglected, in such cases the effect of viscosity is negligible and f' depends upon relative roughness of the pipe.

Colebrook equation can also be written as $\frac{1}{\sqrt{f'}} = -0.86\ln\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f'}}\right)$

MINOR LOSSES

Those losses which occur in pipelines due to bends, elbows, joints, valves, inlet, outlet, expansion, contraction etc are called minor losses. This is a misnomer because in many cases/situations they are more important than the losses due to pipe friction.

In general when $\frac{L}{D}$>2000, velocity head and minor losses should be neglected in the Bernoulli's equation.

CHAPTER TWO: SOLUTION OF SIMPLE PIPE FLOW PROBLEMS

The 3 simple pipe flow cases that are basic to solutions of the more complex problems are:

(i) **Given:** Discharge, Diameter, Length, Coefficient of dynamic/absolute of Kinematics' Viscosity, and absolute rough ness and **required to find Head loss due to friction**. (i.e. given Q, D, L, $\mu, \upsilon, \varepsilon$ and **required to find h_f**)

(ii) **Given:** $h_f, L, D, \mu, or\ \upsilon, \varepsilon$ **required to find Q**

(iii) Given: $h_f, Q, L, \mu, or, \upsilon, \varepsilon$ **required to find D**

CASE 1: EXAMPLE 1

(i) Calculate the loss head due to friction and the power required to maintain flow in a horizontal circular pipe 40mm diameter and 750m long when water with coefficient of dynamic viscosity equals $1.14x10^{-3}\ \frac{N.s}{m^2}$, flows at (a) 4liter/minute (b) 30Liter/minute. Assume that for the pipe the absolute roughness is $8x10^{-5}m$.

SOLUTION

- Establish whether the flow is Laminar or Turbulent:

$$\mathrm{Re} = \frac{\rho dv}{\mu} = \frac{vd}{\upsilon}$$

$$Note: \upsilon = \frac{\mu}{\rho}$$

$$Q = \frac{4x10^{-3}}{60} = 6.67x10^{-5} m^3 / s$$

$$A = \frac{\pi D^2}{4} = 1.26x10^{-3} m^2$$

$$V = \frac{Q}{A} = 0.053 m / s$$

$$\mathrm{Re} = \frac{\rho vd}{\mu} = \frac{10^3 x0.053x0.04}{1.14x10^{-3}} = 1862$$

- The flow is Lamina Re<2000
- For Laminar flow the friction factor can be calculated thus: $f' = \frac{64}{\mathrm{Re}} = 0.03436$
- Head loss due to friction, $h_f = f'\frac{LV^2}{D2g}$ normally referred to as Darcy Weisbach formula/equation
- $\therefore h_f = f'\frac{LV^2}{D2g} = 0.092m$
- Power required to maintain flow $P = \rho g h_f Q = \gamma H_f Q = 10^3 x 9.81 x 0.092 x 6.67 x 10^{-5} = 0.06 Watts$

$$Q = \frac{30x10^{-3}}{60} = 5x10^{-4} m^3 / s$$

(ii) $V = \frac{Q}{A} = 0.4 m/s$

$$\mathrm{Re} = \frac{\rho VD}{\mu} = 14{,}035 = 1.4x10^4$$

$$\mathrm{Re} > 2000$$

The flow is Turbulent

Calculate the relative roughness $\frac{\varepsilon}{D} = \frac{8x10^{-5}}{0.04} = 0.02$

Use Moody's Chart for $\mathrm{Re} = 1.4x10^4$ **and** $\frac{\varepsilon}{D} = 0.02$

$$f' = 0.032$$

$$h_f = 4.89m$$

$$Power = 24.0 Watts$$

CASE 2 EXAMPE 2

2. Water at 15^0C flows through a 30cm diameter riveted steel pipe, absolute roughness of 3mm, with head loss of 6m in 300m. Determine the flow.

SOLUTION

$$\frac{\varepsilon}{D} = \frac{0.003}{0.3} = 0.01$$

Assume $f' = 0.04$

$$h_f = \frac{f' L V^2}{d 2g}$$

$$6 = 0.04 \frac{300}{0.3} \frac{V^2}{19.62}$$

$$V = 1.715 m/s$$

From table of physical properties of water (in any standard text book SI units) at 15^0C , Kinematic Viscosity is $1.139x10^{-6} m^2/s$.

$$\therefore \mathrm{Re} = \frac{Vd}{\upsilon} = 451712 \cong 4.5x10^5$$

from Mood's Chart for $\frac{\varepsilon}{D} = 0.01$ and $\mathrm{Re} = 4.5x10^5$

$f' = 0.038$ (this value is close enough to the assumed value) it is okay.

$$Q = AV = \pi r^2 \sqrt{\frac{h_f d 2g}{f' L}} = \pi \left(r^2\right) \sqrt{\frac{6x0.3x19.62}{0.038x300}} = 0.1245 m^3/s$$

CASE 3 EXAMPLE 3

In the third case with **<u>Diameter</u>** unknown:

(i) There are 2 unknowns in the Darcy-Weisbsch equation f', V and D. $h_f = \dfrac{f' L V^2}{d 2g}$, f', V, d unknown

(ii) There are 2 unknowns in the continuity equation V and d.

(iii) There are 3 unknowns in Reynolds Number equation V, D, Re

(iv) The relative roughness is also unknown

<u>SOLUTION</u>

Using the continuity equation to eliminate the **<u>velocity</u>** in darcy-Weisbach equation and in the expression for **Re** the problem will be simplified.

$$V = \frac{Q}{A}$$

$$h_f = f' \frac{L}{D} \frac{Q^2}{2g\left(\frac{\pi D^2}{4}\right)^2} \cdots\cdots\cdots\cdots 1$$

$$D^5 = \frac{8LQ^2}{h_f g \pi^2} f' = C_1 f'$$

Where $C_1 =$ the known quantities $= \dfrac{8LQ^2}{h_f g \pi^2}$

But $VD^2 = \dfrac{4Q}{\pi} \cdots\cdots\cdots$ continuity equation

$$\mathrm{Re} = \frac{VD}{\upsilon} = \frac{4Q}{\pi \upsilon D} = \frac{C_2}{D} \cdots\cdots\cdots\cdots 2$$

$$C_2 = known quantities = \frac{4Q}{\pi \upsilon}$$

The solution is now effected by the following procedure:

(i) Assume a value of f'

(ii) Solve equation 1 for D

(iii) Solve equation 2 for Re

(iv) Find the relative roughness

(v) Find new f' from moody's chart with the $\left(\mathrm{Re}, \frac{\varepsilon}{D}\right)$

(vi) Use the new f' and repeat procedure

(vii) When the value of f' does not change in the first two significant figures all equations are satisfied and the problem is solved.

<u>EXAMPLE 3</u>

4. Determine the size of clean wrought iron pipe required to convey 260L/s of oil of kinematic viscosity of $9.26x10^{-6}m^2/s$, and 3048m length, with head loss of 22.8m. Absolute roughness is 0.046mm.

<u>SOLUTION</u>

$$D^5 = \frac{8LQ^2 f'}{h_f g\pi^2} = 0.745 f'$$

$$\mathrm{Re} = \frac{4Q}{\pi \upsilon D} = \frac{35634}{D}$$

Assume f'=0.02

$$D = 0.431m$$

$$\frac{\varepsilon}{d} = 1.067x10^{-4}$$

$$\mathrm{Re} = 8.268x10^4$$

From Moody's chart f'=0.019

$$\frac{\varepsilon}{D} = 0.00011$$

$$\mathrm{Re} = 83{,}451$$

$$f' = 0.019$$

f' doesn't change significantly

2^{nd} trial for f'=0.019

CHAPTER THREE: MULTIPLE PIPE SYSTEMS ANALYSIS

3.1 Pipe in series: Discharge is constant i.e. Q=constant

The diagram and illustration as discussed in the class

$$h_f = \frac{f_1' L_1}{d_1}\frac{V_1^{\,2}}{2g} + \frac{f'_2\, L_2 V_2^{\,2}}{d_2\, 2g} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$assume,\ f'_1 = f'_2 = f,\ thesame$$

Substitute

$$V = \frac{Q}{A}$$

$$\therefore h_f = \frac{f'_1\, L_1}{2g}\frac{16Q^2}{\pi^2 d^5{}_1} + \frac{f'\, L_2 16Q^2}{2g\pi^2 d_2^{\;5}} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

But $\frac{f' LQ^2 x16}{19.62\pi^2 d^5} = \frac{f' LQ^2}{12d^5} = rQ^2$

Where r=pipe constant= $\frac{f' L}{12d^5}$

$$\therefore h_f = r_1 Q^2 + r_2 Q^2 + r_3 Q^2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$h_f = Q^2 \sum_1^n r$$

or

$$Q = \sqrt{\frac{h_f}{\sum r}}$$

3.2 Equivalent Pipe Method for pipe in series:

An equivalent pipe is a pipe which will carry this particular flow rate and produce the same head loss as two or more pipes. If we are to replace this complex system with a single equivalent pipe;

$h_f = r_e Q^2$ where r_e=pipe constant for equivalent pipe

Hence in a series pipe system

$$r_e = \sum_1^n r$$

$$r_e Q^2 = Q^2 \sum_1^n r$$

3.3 Pipes in parallel: Head loss is a constant i.e. h_f=constant

The diagram and illustration as discussed in the class

$h_{f1} = h_{f2} = h_{f3}$

The head loss in each pipe between junctions where parallel pipes part and join again must be equal. $Q_T = Q_1 + Q_2 + Q_3$. The total flow rate will equal the s um of individual flow rates. $Q_T = \sqrt{\frac{h_f}{r_1}} + \sqrt{\frac{h_f}{r_2}} + \sqrt{\frac{h_f}{r_3}}$

$$Q_T = \sqrt{h_f}\sum_{1}^{n}\left(\frac{1}{\sqrt{r}}\right)$$

3.4 Equivalent Pipe Method for pipe in parallel

If we want to replace the system with a single equivalent pipe then: $h_f = r_e Q_T{}^2$

$$Q_T = \sqrt{\frac{h_f}{r_e}}$$

$$r_e = \left(\frac{1}{\sum_{1}^{n}\left(\frac{1}{\sqrt{r}}\right)}\right)^2$$

or

$$r_e = \frac{1}{\left(\sum_{1}^{n}\frac{1}{\sqrt{r}}\right)^2}$$

EXAMPLE 4: For pipe in series Q=constant.

Pipe in series as shown on the board. Find Q? Given total head loss as 26m, f'=0.01 kc=0.33, where kc is the coefficient of contraction. Consider all losses and use equivalent pipe method.

SOLUTION

(i) Consider all losses : Write Bernoulli's Equation from reservoir A to B

H_T=Entrance loss +head loss due to friction+ head loss due to contraction +head loss due to friction + Exit loss

$$H_T = \frac{0.5v_1^2}{2g} + f'\frac{Lv_1^2}{d2g} + \frac{0.33v_2^2}{2g} + f'\frac{Lv_1^2}{d2g} + \frac{v_2^2}{2g}$$

$$26 = 0.225v_1^2 + 0.468v_2^2$$

$$V_2 = V_1\left(\frac{A_1}{A_2}\right) = V_1\left(\frac{d_1^2}{d_2^2}\right) = 4V_1$$

$$26 = 0.225V_1^2 + 0.468(4V_1)^2 = 7.71V_1^2$$

$$V_1 = 1.83m/s$$

$$Q = A_1V_1 = A_2V_2 = 0.14m^3/s$$

(ii) Using equivalent pipe method

Neglecting minor losses and calculate pipe constants

$$r_1 = \frac{f'_1 L_1}{12d_1^5} = \frac{0.01x122}{12(0.31)^5} = 35.51$$

$$r_2 = 1136.37$$

For pipe in series $r_e = \sum_1^2 r = 35.51 + 1136.37 = 1171.88$

$$h_f = r_eQ^2$$

$$Q = \sqrt{\frac{h_f}{r_e}} = \sqrt{\frac{26}{1171.88}} = 0.149m^3/s \cong 0.15m^3/s$$

Example 5 for pipe in parallel h_f=constant

Find the head loss across the system shown and discharges in each pipe.

SOLUTION

$$r = \frac{f'L}{12d^5}$$

D(mm)	r	$\sqrt{r}$	$\frac{1}{\sqrt{r}}$
305	785.8	28.03	0.036
200	3812.5	61.75	0.016
405	260.0	16.12	0.062
$\sum$			0.114

$$r_e = \left(\frac{1}{\sum \frac{1}{\sqrt{r}}}\right)^2 \text{ or } \frac{1}{\left(\sum \frac{1}{\sqrt{r}}\right)^2}$$

$$= \left(\frac{1}{0.114}\right)^2 \text{ or } \frac{1}{(0.114)^2} = 76.95$$

$$r_e = 76.95$$

$$h_f = 76.95(0.34)^2 = 8.9m$$

(ii) To find the discharge in individual pipes, you have to consider individual pipe

$$h_f = \frac{f'LV^2}{d2g} = \frac{0.017x1464xV^2}{305x2x9.81} = 8.9m$$

$$V = 1.46m/s$$

Consider 305 mm diameter pipe $Q_{305} = 0.107m^3/s$

$$Q_{200} = 0.049m^3/s$$

$$Q_{405} = 0.186m^3$$

$$Q_T = (Q_{305} + Q_{200} + Q_{405}) \cong 0.34m^3/s$$

Using Equivalent pipe method

h_f=constant

$$h_f = r_e Q_T{}^2$$

$$r_e Q_T{}^2 = r_1 Q_{305}{}^2$$

$$76.95(0.34)^2 = 785.8 Q_{305}{}^2$$

$$Q_{305} = 0.106 m^3 / s$$

$$Q_{200} = 0.048 m^3 / s$$

$$Q_{405} = 0.185 m^3 / s$$

$$Q_T = 0.339 \cong 0.34 m^3 / s$$

EXERCISES

1) Two water reservoirs are connected by a pipe 610m of 0.3m diameter, f'=0.038 and the flow produced by the difference in water surface elevations equals 0.17cumecs, if a new pipe of 0.3m diameter and length 460m is laid from the highest reservoir parallel to the old line and connected to the old line 460m from its inlet. Determine the total discharge for the improved system, f'=0.019 for the new pipe. Neglect secondary losses.
2) One of the advantages of parallel connections in water pipe network distribution is to enhance discharge. Demonstrate this fact with this question. A straight 300mm diameter pipeline 5km long is laid between two reservoirs of surface elevations150m and 100m. The pipeline enters these reservoirs 10m below their water surface levels. To increase the capacity of the line a 300mm diameter line 2.5km long is laid parallel to and from the original lines mid-point to the lower reservoir. What increase in flow rate is gained by installing the new line? Assume the friction factor is 0.02 for all the pipes and neglect minor losses.
3) A three pipe system is such that the total pressure drop is 1.5bar and the elevation drop is 5m. The length L, diameter d and friction factorf' for the three pipes are given in the table below.

Pipe	Length L (m)	Diameter D (m)	f’
1	150	10.0	0.0275
2	200	7.5	0.0245
3	75	5.0	0.0315

Calculate the ratio of the total flow rates for the case in which the pipes are connected in series compare to the case in which the pipes are in parallel. The density of water can be taken as 1000kg/m^3.

4) Two long pipes are used to convey water between two reservoirs whose water surfaces are at different elevations. One pipe has a diameter twice that of the other. If both pipes have the same value of friction factor and if minor losses are neglected, what is the ratio of the flow rates through the two pipes

5) A 2.0m diameter concrete pipe of length 1560m for which ε=1.5mm conveys 12^{0}C water between two reservoirs at a rate of 8.0m^3/s. What must be the difference in water surface elevation between the two reservoirs?

6) For the diagram below and the information in the table below.

Pipe No	Diameter (mm)	Length (m)	f’
1	200	300	0.021
2	300	300	0.0185
3	450	300	0.0165
4	300	600	0.0185
5	300	700	0.0185

Find the equivalent length of a 300mm diameter clean cast iron pipe to replace the above system. For H=10m, ε=0.25mm, what is Q?

7) (a) For laminar flow in pipes $f' = \frac{64}{Re}$. Using this information, develop the expression for the velocity in terms of lost head due to friction, diameter and other pertinent items.

(b) How much power is lost per meter of pipe length when oil with a viscosity of 0.20N.s/m^2 flows in a 20cm diameter pipe at 0.5L/s. The oil has a density of 840kg/m^3.

(c) Oil of absolute viscosity 0.1Pa.s and relative density 0.85 flows through 3048m of 305mm cast iron pipe at the rate of 44.4x$10^{-3}$$m^3$/s. What is the lost head in the pipe?

8) Water is pumped 15Km, from a reservoir at elevation 30m to second reservoir 64m. The pipeline connecting the reservoirs is 1.5m in diameter. It is concrete and has an absolute roughness of 0.9mm. If the flow is 109L/s and pumping station efficiency is 80%, what will be the monthly power bill if electricity costs 30kobo per kilowatt hour? (f'=0.0175).

9) It is necessary to pump 0.38m^3/s of water from reservoir at an elevation of 270m to a tank whose bottom is at an elevation of 330m. The pumping unit is located at elevation 270m. The suction pipe is 0.6m in diameter and very short so head losses may be neglected. The pipeline from the pump to the upper tank is 123m long and is 0.5m in diameter. Consider the minor losses in the line to equal 0.75m if water. Find the maximum lift of the pump and the power required for pumping if the pump efficiency is 76%. The maximum depth of water in the tank is 11.4m and the supply lines are cast iron, f'=0.017.

CHAPTER FOUR: EMPIRICAL EQUATION

The most widely used is the HAZEN WILLIAMS equation:

$$Q = 0.2785Cd^{2.63}S^{0.54}$$
$$Q = 0.849CAR^{0.63}S^{0.45}$$

Q=m^3/s =discharge

C=Hazen Williams roughness coefficient

D=diameter (m)

S= Slope of the energy line=h_f/L

R=A/P= Hydraulic Radius

$$H_f = \left(\frac{10.7L}{C^{1.852}D^{4.87}}\right)Q^{1.852} = rQ^{1.852}$$

Pipe in series $h_f = r_e Q_T{}^{1.852}$

Pipe in parallel $h_f = Q_T{}^{1.852}$

EXAMPLE

a) Two parallel pipes each 150m long, one 200mm diameter and the other 150mm diameter, each with C=120 and Q_T=0.14m^3/s, determine the head loss in meter of water.

b) Two pipe in series one 30m long with a 300mm diameter and the second 100m long with a 250mm diameter each having a C=110, Q_T=0.14m^3/s, determine the head loss in meter of water.

SOLUTION: Pipe in parallel

(a) $H_f = \left(\frac{10.7L}{C^{1.852}D^{4.87}}\right)Q^{1.852} = rQ^{1.852}$

but r_1=579.4, r_2=2341.9

$$\left(\frac{1}{r_e}\right)^{0.54} = \left(\frac{1}{579.4}\right)^{0.54} + \left(\frac{1}{2351.9}\right)^{0.54} = 0.0473$$

$$r_e = \left(\frac{1}{0.0473}\right)^{1.852} = 284.54$$

$$h_f = r_e Q_T{}^{!.852} = 7.49m$$

$$r_e = \left(\frac{1}{\sum\left(\frac{1}{r}\right)^{1/n}}\right)^{n}$$

NOTE: $n = 1.852$

$$h_f = r_e Q_T{}^{1.852}$$

(b) Pipe in series

$$r_e = r_1 + r_2 = 18.8961 + 153.05 = 171.95$$

$$h_f = r_e Q_T{}^{1.852} = 4.5m$$

<u>TAKE HOME ASSIGNMENT</u>

(1) The dimensions of the figure shown below are shown in this table,

Pipe	L(m)	D(m)	C	r
1	75	0.05	110	2.91x10^5
2	100	0.07	110	5.39x10^4
3	150	0.1	100	2.37x10^4

Find the total discharge in reservoir B.

(2) Water flows in the parallel pipe system shown below for which the following data are available.

Pipe	Diameter (m)	Length (m)	f'
AaB	0.1	300	0.024
AbB	0.15	250	0.022
AcB	0.2	500	0.02

The supply pipe to point A is 0.3m diameter and the mean velocity of water in it is 3m/s. If the elevation of point A is 100m and elevation of point B is 30m above datum, calculate the pressure at point B if that at point A is 200KN/m^2. What is the discharge in each pipe, neglect all minor losses.

BRANCHING PIPES

The three interconnected reservoirs as shown above;

1) Flow through each pipe is wanted
2) Reservoir elevations are given with the sizes and types of pipes
3) Fluid properties are assumed known
4) The Darcy-Weisbach and continuity equation must be satisfied for each pipe.
5) The flow into the junction (J) must be equal to the flow out of the junction

SOLUTION PROCEDURE: Solution is effected thus:

(i) Assume an elevation of the Hydraulic Grade Line (HGL) at the junction

(ii) Compute Q_1,Q_2 and Q_3

(iii) Substituting into the continuity equation i.e. $Q_1 = Q_2 + Q_3$ or

$Q_1 + Q_2 = Q_3$

If the flow into the junction is to great (more), a higher grade-line elevation which will reduce the inflow and increase the outflow is assumed. $Z = y + \frac{P}{\gamma} = HGL$

Z=Position of hydraulic grade line or piezeometric height.

ANALYSIS

NOTE: It is supposed that all pipes are sufficiently long, so that minor losses and velocity heads may be neglected. (When L/d>2000 neglect minor losses)

$$Z = y + \frac{P}{\gamma} = HGL$$

$$h_f = h - Z = rQ^2$$

Write flow equations for all the three pipes

(i)
$$h_1 - Z = r_1 Q_1{}^2$$
$$Q_1 = \sqrt{\frac{|h_1 - Z|}{r_1}}$$

(ii)
$$h_2 - Z = r_2 Q_2{}^2$$
$$Q_2 = \sqrt{\frac{|h_2 - Z|}{r_2}}$$

(iii)
$$h_3 - Z = r_3 Q_3{}^2$$
$$Q_3 = \sqrt{\frac{h_3 - Z}{r_3}}$$

Equation of continuity $Q_1 = Q_2 + Q_3$ **or** $Q_1 + Q_2 = Q_3$

Sign convection must be adopted and maintained (signs of flow are dictated by choice of h-z or z-h)

- Towards joint positive +
- Away from joint negative –
- $\therefore \sum_1^n Q = 0$

If Z is first estimated and sum of Q calculated, it will result in a value that sum of Q will not equal to zero i.e. . Where dQ is a function of the error in the estimated value of Z. If δQ is very small then $\frac{\delta Q}{\delta Z} = -\sum_1^n \frac{Q}{2(h-Z)}$ the error in the estimated value of Z is δZ . $\therefore \delta Z = \frac{-2\delta Q}{\sum \frac{Q}{h-z}}$

Thus the "correction" to apply to Z (assumed) to make sum of Q to zero is

$$+\frac{2\delta Q}{\sum \frac{Q}{h-z}}$$

PROCEDURE FOR ANALYSIS

(1) Assume an initial value for Z

(2) Compute resulting $\sum Q \neq 0$

(3) If appreciable, estimate $\sum \frac{Q}{h-z}$ correct z with the error function and recalculate.

EXAMPLES

(1) A reservoir A with its surface 60m above datum supplies water to a junction D through a 300mm diameter pipe 1500m long. From the junction, a 250mm diameter pipe 800m long feeds reservoir B, in which the surface level is 30m above datum, while a 200mm diameter pipe 400m long feeds reservoir C, in which the surface level is 15m above datum. Calculate the volume rate of flow to each reservoir. Assume the loss of head due to friction is given by

$h = \dfrac{f'LQ^2}{12d^5}$ and the friction factor for each pipe is 0.04.

(2) Given the information below on the diagram for four reservoirs. The elevation of junction is 8m and f' for all the pipes is 0.02.

Pipes	h(m)	L(m)	D(m)	$r = \dfrac{f'L}{12d^5}$
1	17	150	0.6	3.215
2	10	300	0.3	205.48
3	6	900	0.45	81.45
4	3	450	0.3	308.22

(i) Calculate the discharge Q to each reservoir.

(ii) Calculate the pressure in pipeline at joint.

(3) Three open reservoirs A,B and C have constant water surface elevations 90m, 45 and 72m respectively. The reservoirs are connected through a common junction J by pipe lines having characteristics given below. The junction J is at elevation 60m. Determine the flow in the pipes.

Pipe	Length (m)	Diameter (m)	f’
AJ	450	0.45	0.0075
BJ	600	0.3	0.01
CJ	300	0.3	0.0075

(4) Find the discharges for the system tree reservoirs with the following pipe data and reservoir elevations

L1=3000 m	D1=1m	f’=0.014	h=30m
L2=600m	D2=0.45m	f’=0.024	h=18m
L3=1000m	D3=0.6m	f’=0.02	h=9m

PIPE NETWORKS Flow in a water distribution network however complicated must satisfy the basic relations of continuity and energy.

Diagram (Figure)

CRITERIA

(i) Sum of discharges at a node is zero, i.e. the flow into any junction or node must equal the flow out of it (continuity equation)

(ii) Sum of all head losses around a closed circuit must be zero

(iii) The flow in each pipe must satisfy the pipe friction laws (Darcy Weisbach or equivalent exponential friction formula) for flow in a single pipe

Since it is complicated to solve network problems analytically, methods of successive approximation are utilized.

HARDY CROSS METHOD

The Hardy-Cross method is one in which flows are assumed for each pipe so that continuity is satisfied at every junction. A correction to the flow in each circuit is the computed in turn and applied to bring the circuits into closer balance.

From Figure:

(a) Main Circuit

$$r_1Q_1^{\ 2} + r_2Q_2^{\ 2} + r_3Q_3^{\ 2} - r_4Q^2{}_4 - r_5Q_5^{\ 2} = 0$$

(b) Sub Circuit 1

$$r_1Q_1^{\ 2} + r_6Q_6^{\ 2} - r_5Q_5^{\ 2} = 0$$

(c) Sub-Circuit 2

$$r_2 Q_2^{\ 2} + r_3 Q_3^{\ 2} - r_4 Q_4^{\ 2} - r_6 Q_6^{\ 2} = 0$$

PROCEDURE FOR ANALYSIS

(i) Assume an initial (trial) value for each discharge (Q_a) bearing in mind criteria 1 i.e. $\sum_1^n Q = 0$

(ii) Compute the corresponding value $h_{fa} = rQ_a^{\ 2}$

(iii) Determine the algebraic sum of all head losses in each closed circuit. (Normally not equal to zero).

(iv) Compute values of $\sum\left(\frac{h_{fa}}{Q_a}\right)$ for each closed circuit

(v) Determine the correction to the assumed values of Q_a to be applied to each closed circuit. Using $\Delta Q = \frac{-\sum h_{fa}}{2\sum\left(\frac{h_{fa}}{Q_a}\right)}$

(vi) Revise flows in each pipe by $Q = (Q_a + \Delta Q)$

Repeat from (ii) until $\sum h_f = 0$ in all circuits.

<u>NOTE:</u> The derivation of ΔQ expression could be checked from any advanced text on this subject.

SIGN CONVECTION

In allocating signs to the discharges move around each closed circuit in a clockwise direction given all flows in a clockwise direction positive sign (+ve) and all flows opposing this a negative sign (-ve).

When computing $h_{fa} = rQ_a^{\ 2}$ use the form $h_{fa} = Q_a\,|Q_a\,|$ to preserve the negative sign when present, by inspection, it can be seen that when the flow direction is reversed in a pipe, the direction of the slope of hydraulic gradient is also changed.

EXAMPLE

Water enters the four sided ring min shown below at A at the rate of $0.4m^3/s$ and is delivered at B, C and D at the rate of 0.15,0.10 and 0.15 m^3/s. All pipes are 0.6m in diameter with a friction coefficient of 0.0132 and their lengths are AB and CD 150m, BC 300m and DA 240m. Determine the flow through each pipe and the pressures at B, C and D if that at A is $105KN/m^2$.

NOTE:

1. ALL PROBLEMS AND EXERCISES WILL BE SOLVED IN THE CLASS AND SOME WILL BE TAKEN AT TUTORIAL CLASS
2. THIS CLASS NOTE WILL NOT REPLACE THE RECOMMENDED TEXTS
3. SOME OF THE BOOKS ARE AVAILABLE IN THE MAIN LIBRARY AND COLLEGE LIBRARY