UNIVERSITY OF AGRICULTURE, ABEOKUTA

B.Sc Degree Examination

2009/2010

SECOND SEMESTER EXAMINATIONS

MATHEMATICS

MTS 232: ORDINARY DIFFERENTIAL AND DIFFERENCE EQUATIONS

Thursday, October 14, 2010 . Time Allowed : 3 Hours .

Attempt **ANY THREE** QUESTIONS.

1(a) State a necessary and sufficient condition for the following differential equation to be exact:

$$P(t,x)\,dt + Q(t,x)\,dx = 0. \tag{1.1}$$

Determine if the o.d.e.

$$x \sin t\,dt + \cos t\,dx = 0 \tag{1.2}$$

is exact and obtain its solution.

1(b) Write down an integrating factor for the o.d.e.

$$\frac{dx}{dt} + a_0(t)\,x = a_1(t). \tag{1.3}$$

Hence or otherwise, solve

$$\cot t\,\frac{dx}{dt} + x = \cos t. \tag{1.4}$$

2(a) Given the primitive function

$$x = a\,e^t + b \sin t \tag{2.1}$$

where a and b are arbitrary constants, find the differential equation associated with it.

2(b) If the movement of a particle is described by the o.d.e.

$$\frac{dv}{dt} = \frac{p}{R - p^2\,v} \tag{2.2}$$

where R and p are known constants,
deduce that

$$t = \frac{R\,v}{p} - \frac{1}{2}\,p\,v^2 \tag{2.3}$$

if $v = 0$ when $t = 0$.

3(a) Solve the following o.d.e.

$$\frac{dx}{dt} = \frac{2\,t\,x + 3\,x^2}{t^2 + 2\,t\,x} \tag{3.1}$$

3(b) If an object is heated to 300°F and allowed to cool in a room where the ambient temperature is 80°F. What will be its temperature after 20 minutes if after 10 minutes, the temperature is 250°F ? Hint : Employ Newton's law of cooling described by the o.d.e.

$$\frac{dU}{dt} = -K\,(U(t) - U_0) \tag{3.2}$$

where $U(t)$ is the temperature of body at time t and U_0 is the constant temperature of the surrounding medium with K being a positive constant.

4(a) Find the two linearly independent solutions of the homogeneous o.d.e. associated with

$$x'' + x = \tan t. \tag{4.1}$$

Use the method of variation of constants to obtain the particular solution of (4.1) and then write down its general solution.

4(b) Show that the o.d.e.

$$t\,\frac{d^2x}{dt^2} = 2\left[\left(\frac{dx}{dt}\right)^2 - \frac{dx}{dt}\right] \tag{4.2}$$

is of the form

$$F(t\,, x'\,, x'') = 0. \tag{4.3}$$

Solve (4.3) by using the transformation

$$v = \frac{dx}{dt}.$$

5(a) Given that

$$u_r = P\,4^r \tag{5.1}$$

where P is an arbitrary constant, obtain a first order difference equation from (5.1).

5(b) Solve the difference equation

$$u_{r+1} - a_r\,u_r = 0. \tag{5.2}$$