UNIVERSITY OF AGRICULTURE, ABEOKUTA

B.Sc Degree Examination

2009/2010

SECOND SEMESTER EXAMINATIONS

MATHEMATICS

MTS 242: Mathematical Methods

Tuesday, October 5, 2010. Time Allowed : 2 1/2 Hours.

Attempt **ANY Four** questions.

1(a) Show using the definition of a limit that

$$\lim_{(x,y)\to(5,7)} (3x + 2y) = 29.$$

1(b) Show that

$$\lim_{(x,y)\to(0,0)} \frac{y^2 - x^2}{x^2 + y^2}$$

does not exist.

2(a) Find the domain of the function

$$f(x, y) = \frac{\sqrt{x - y}}{x + y}$$

2(b) Let $f(x, y) = 3xy^2 - 2x^2y$
then find both partial derivatives f_x and f_y.

3(a) If a particle is falling in a fluid, then according to stoke's law, the velocity of the particle is given by

$$V = \frac{2g}{9}(\rho_p - \rho_f)\frac{r^2}{\nu},$$

where g is the acceleration due to gravity, $\rho_p =$ density of particle, $\rho_f =$ density of fluid, $r =$ radius of particle and $\nu =$ the absolute viscosity of the liquid.
Calculate V_{ρ_p}, V_{ρ_f}, V_r, V_ν.

3(b) Find the Taylor series expansion of $\cos x$ about the point $a = 2\pi$.

4(a) Use the binomial series to estimate $\sqrt{1.25}$ with an error of less than 0.001.

4(b) In each of the following problems ((i) through (iii)), a, b, and c refer to the equation $f(b) - f(a) = (b - a)f'(c)$, which expresses the Mean Value Theorem. Given $f(x)$, a, and b, find c.

(i) $f(x) = x^2 + 2x - 1;\ a = 0,\ b = 1$

(ii) $f(x) = x^3;\ a = 0,\ b = 3$

(iii) $f(x) = x^{\frac{2}{3}};\ a = 0,\ b = 1$

5(a) Find the volume of the solid whose base is in the $xy-$ plane and is the triangle bounded by the $x-$ axis, the line $y = x$, and the line $x = 1$, while the top of the solid is in the plane.
$z = f(x, y) = 3 - x - y$.

5(b) Find the polar moment of inertia about the origin of a thin plate of density $\delta = 1$ bounded by the circle $x^2 + y^2 = 1$.

6(a) If $\mathbf{F} = y\mathbf{i} + x\mathbf{j}$, evaluate the line integral $\int_A^B \mathbf{F}.\, d\mathbf{R}$ along the straight line from $A(1,1,1)$ to $B(3,3,3)$.

6(b) Use Green's theorem to find the area enclosed by the ellipse
$x = a\cos\theta,\ y = b\sin\theta, 0 \leq \theta \leq 2\pi$, where $M = y$ and $N = x$.