UNIVERSITY OF AGRICULTURE ABEOKUTA

Department of Mathematics

B.Sc Examinations

MTS 443: Mathematical Methods III

2009/2010 First Semester Examinations

TIME ALLOWED: 3 Hours

Instruction: Answer any 2 questions from each section.

SECTION A

1. a.(i) Let $\{\phi_n\}$ $(n = 1, 2, 3, \ldots)$ be an infinite set of functions on the interval $a \leq x \leq b$. When is $\{\phi_n\}$ said to be an (α) orthogonal system, (β) orthonormal system with respect to the weight function $r(x)$ defined on $a \leq x \leq b$.

   (ii) Show that if the functions $g_1(x), g_2(x), \ldots$ form an orthogonal set on $a \leq x \leq b$, then the functions $g_1(ct+k), g_2(ct+k), \ldots$ where $c > 0$ form an orthogonal set on the interval $\frac{a-k}{c} \leq t \leq \frac{b-k}{c}$.

   (b) Let $\{P_n\}$, $n = 1, 2, 3, \ldots$ be an infinite set of polynomial functions such that

   (i) P_n is of degree n, $n = 0, 1, 2, 3, \ldots$

   (ii) $P_n(1) = 1$, $n = 0, 1, 2, 3, \ldots$ and,

   (iii) the set $\{P_n\}$ is an orthogonal system with respect to the weight function r such that $r(x) = 1$ on the interval $-1 \leq x \leq 1$.

   Construct consecutively the members P_0, P_1, P_2 and P_3 of this set by writing

   $P_0(x) = a_0,$

   $P_1(x) = b_0 x + b_1,$

   $P_3(x) = c_0 x^2 + c_1 x + c_2.$

   $P_4(x) = d_0 x^3 + d_1 x^2 + d_2 x + d_3,$

   and determining the constraints in each expression so that it has the value 1 at $x = 1$ and is orthogonal to each of the preceding expressions with respect to r on $-1 \leq x \leq 1$.

2. a. Given the second order differential equation

$$a(x)y'' + b(x)y' + c(x)y = f(x). \tag{2.1}$$

   Obtain the self-adjoint form

$$\left(p(x)\frac{dy}{dx}\right)' + q(x)y = F(x) \tag{2.2}$$

   from (2.1). Hence or otherwise resolve the Legendre's differential equation

$$(1 - x^2)y'' - 2xy + n(n+1)y = 0$$

   in the form (2.2).

   b. Show that the Legendre's polynomials, $P_n(x)$ are orthogonal with respect to the weight function, $w(x) = 1$ over the interval $[-1, 1]$.

3. a. Find the eigenvalues and eigenfunctions of the given boundary value problem

$$y'' + \lambda y = 0 \qquad (\lambda > 0)$$

$$y(0) = 0, y'(1) = 0.$$

b. Find the Fourier series of the sawtooth function

$$f(x) = \begin{cases} x+1, & -1 \le x \le 0 \quad ; \\ 1-x, & 0 \le x \le 1 \end{cases} .$$

$$f(x+2) = f(x)$$

Hence determine a series for $\frac{\pi^2}{8}$.

SECTION B

4. a. Define Fourier integral transform for a function $f(x)$.

b. Solve the boundary value problem

$$y''(x) - k^2 y(x) = -f(x), \quad -\infty < x < \infty$$

where k is a constant and $f(x)$ is specified such that $y(x), y'(x) \to 0$ as $|x| \to \infty$ using Fourier Integral transform.

c. Obtain the solution for the other form of problem in (b) written as

$$y''(x) + k^2 y(x) = -f(x), \quad -\infty < x < \infty.$$

5. a. State without proof the convolution theorem for Fourier transforms.

b. Given the Cauchy problem for wave equation

$$U_{tt} - c^2 U_{xx} = 0, \quad -\infty < x < \infty, t > 0$$

where

$$U(x,0) = f(x), \quad -\infty < x < \infty$$

$$U_t(x,0) = g(x), \quad -\infty < x < \infty.$$

Show that

$$U(\lambda, 0) = F(\lambda) \text{ and } \frac{\partial U(\lambda,0)}{\partial t} = G(\lambda).$$

c. Obtain the complete solution of the problem in b.

6. a. State the Fourier's Integral theorem for the function $f(x)$ and state sufficient conditions for the theorem.

b. Give a brief statement of the Parseval's identity for the functions $f(x)$ and $g(x)$.

c. Given the boundary value problem

$$U_{xx} + U_{yy} = 0, \quad 0 < x < \infty, 0 < y < \infty$$

with the boundary data

$$U(0,y) = 0, U(x,y) \to 0 \text{ as } x \to \infty, \text{ uniformly in } y,$$

$$U(x,\alpha) = 0, U(x,0) = f(x).$$

Show that

$$\frac{\partial^2 U_s(\lambda,y)}{\partial y^2} - \lambda^2 U_s(\lambda,y) = 0.$$