CSC 251-NUMERICAL ANALYSIS I

2 HOUR

ANSWER ANY THREE QUESTIONS

Question 1

(a) If the exact answer is $A$ and the computed answer is $\tilde{A}$, find the relative error
when

(a) $A = 10.147$ \quad $\tilde{A} = 10.159$

(b) $A = 0.0047$ \quad $\tilde{A} = 0.0045$

(c) $A = 0.671 \times 10^{15}$, \quad $\tilde{A} = 0.669 \times 10^{15}$

(b) Use the bisection method to approximate $\sqrt{3}$ to 2 decimal places. Use $f(x) = x^2 - 3$ with $f(0) = -3$ and $f(2) = 1$ as the starting point.

Question 2

(a) Let $a = 0.471 \times 10^{-7}$ and $b = -0.185 \times 10^{-1}$. Use 3 digits floating point arithmetic to compute $a + b, a - b, a \times b$ and $a / b$. Find the rounding error in each case.

(b)(i) Express the numbers $x = 12.74$, $y = 0.0025$ and $z = -12.55$ as three digits floating point numbers.

(ii) Compute the following expression using the three floating point arithmetic.

\[
x - y \\
x + z
\]

(iii) Identify the rounding errors at each step of the calculation, including the representation of $x, y,$ and $z$.

(iv) Calculate the total error due to rounding in the calculation.

Question 3

Sketch the cubic polynomial

\[ f(x) = 4x^3 - 10x^2 + 2x + 5 \]

to get a rough estimate of its roots. Use the Newton Raphson method to approximate each root to 4 decimal places.

(b) With $x$ in radians, show graphically that there are infinitely many solutions of $x = \tan(x)$. 

(ii) Use the Newton Raphson method to find the smallest positive roots.

Question 4

(a) Describe briefly how the Gauss Seidel method differs from the Jacobi Method.
(b) Carry out two iterations of the Jacobi method for solving the following with initial estimate \( x = 0 \).

\[
\begin{bmatrix}
10 & -1 & -1 \\
1 & 10 & -2 \\
3 & 2 & 10
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
=
\begin{bmatrix}
1 \\
0 \\
-2
\end{bmatrix}
\]

Question 5

(a) The Newton's form of Lagrange interpolation expresses

\[
L_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \ldots + a_n(x-x_0)(x-x_1)\ldots(x-x_{n-1})
\]

If one impose the interpolation constraints,

\[
L_n(x_i) = f(x_i) \ for \ i = 0 : n, \ then
\]

\[
f(x_0) = f_0 \Rightarrow a_0 = f_0
\]

\[
f(x_1) = f_1 \Rightarrow a_0 + a_1(x_1-x_0) = f_1
\]

And if \( a_i = \frac{f_i - f_0}{x_i - x_0} \), proof that

\[
a_2 = \frac{x_2-x_0}{x_2-x_1}
\]

\[
a_3 = \frac{x_3-x_0}{x_3-x_1}
\]

Given

\[
L_n(x_3) = f(x_3) = f_3 \Rightarrow a_0 + a_1(x_3-x_0) + a_2(x_3-x_0)(x_3-x_1)
\]

(b) Complete the divided difference table for a function \( f(x) \) giving the divided differences to four decimal places.

<table>
<thead>
<tr>
<th>( i )</th>
<th>( x_i )</th>
<th>( f_i = f[x_i] )</th>
<th>( f[x_i, x_{i+1}] )</th>
<th>( f[x_i, x_{i+1}, x_{i+2}], f[x_i, x_{i+2}, x_{i+3}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0</td>
<td>0.633</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.932</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>2</td>
<td>-0.4</td>
<td>0.389</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>3</td>
<td>0.4</td>
<td>0.783</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>