FST 206: FUNDAMENTALS OF HEAT AND MASS TRANSFER (2 UNITS)

Lecture 1:

Introduction

Heating and cooling are the most common processes found in a food processing plant. Example of food processing operation where heat transfer occurs includes refrigeration, freezing, thermal sterilization, drying, and evaporation. The study of heat transfer is important as it provides a basis on how various food processes involving heat transfer operate. When the fundamentals of heat transfer are well understood, the knowledge can be applied by a food engineer to design appropriate heat transfer equipment and facilities for specific food processes, assess the equipment performance, or improve on the existing process equipment design.

Heat Transfer Theory

There are two ways to view heat transfer by conduction. The first theory explains conductive heat transfer at molecular level. As molecules absorb thermal energy, they vibrate at their respective locator. The amplitude increases with higher thermal energy level. These vibrations are transmitted from one molecule to another without actual translator motion of the molecules.

Another theory on mechanism of conduction states that occurs at molecular level due to movement of the electron which are prevalent in metals. The free electron carries both thermal and electrical conductors. It should be noted that in conductive heat transfer, there is no physical movement of the material. It commonly found in heating or cooling of opaque solid media.

For heat to move from one body to another there must be:

(i) Temperature difference between the two media exchanging heat.
(ii) A medium of allowing the passage of heat between them.

There are 3 basic modes of heat transfer: conduction, convection and radiation.

Conductive Heat Transfer

Generally, the rate of heat transfer is expressed as

$$Rate = \frac{Driving\ force}{Resistance}$$

For conductive heat transfer,

$$Rate = Driving\ force \times Conductance$$

i.e. $\frac{dQ}{dt} = \left[\frac{dT}{dx}\right] \cdot [kA]$

$\frac{dQ}{dt}$ is often simply denoted as q

Therefore, by introducing the negative sign which indicates the flow of heat against temperature gradient

$$q = -k \cdot A \cdot \frac{dT}{dx} \quad \text{or}$$

$$\frac{q}{A} = -k \cdot \frac{dT}{dx} \qquad (1)$$

$\frac{dT}{dx}$ Is the thermal gradient (°C/m)

$\frac{q}{A}$ is the heat flux (W/m^2) across the solid surface

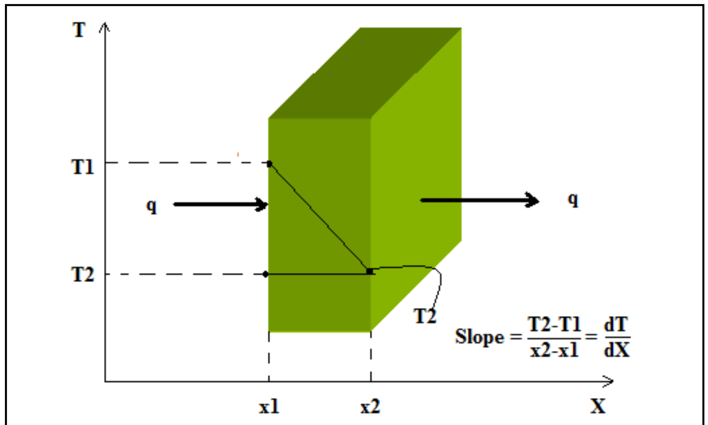


Figure 1: Schematics of conductive heat transfer in a slab

Equation (1) above is also known as Fourier's law of heat conduction. The sign in the eqn (1) indicates that flow from higher temperature region to lower temperature thus satisfying the 2nd law of Thermodynamics. This eqn is applicable when steady state conduction is governing the heat transfer process.

Example 1

One face of a stainless steel plate 1cm thick is maintained at 50^{0}C while the offer is at 20^{0}C. Assuming steady state conductions, calculate the rate of flux through the plate. The thermal conductivity of stainless steel is 17 W/m°C.

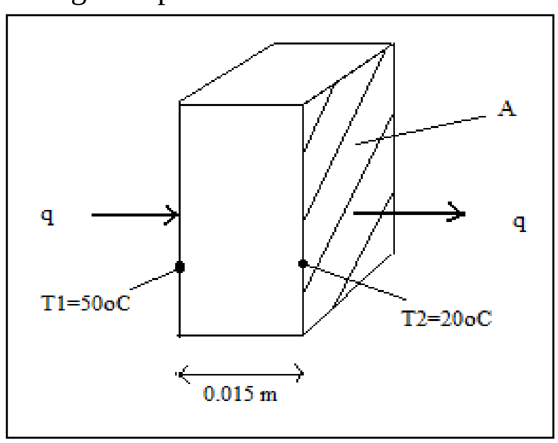
Thickness of plate = 1.5 cm = 0.015 m

dT = T_2-T_1 = (20-50) °C = -30°C

dX = 0.015 – 0 = 0.015m

k = 17 w/m^{0}c

Heat flux $\frac{q}{A} = -k \cdot \frac{dT}{dx}$ = - 17 × (-30)/0.015 = 34,000 W



Example 2

An experiment was conducted to measure thermal conductivity of a formulated food by using a large plane plate of the food material (5mm thick). Under steady state conduction, a temperature difference of 35^{0}C was maintained between the two surfaces of the plate. A heat transfer rate per unit area of 4700 W/m^2 was measured near the centre of either surface. Calculate the thermal conductivity of the product and list two assumptions used in obtaining the result.

Thickness = 5 mm = 0.005 m

dT = 35^{o}C

dx = (0 – 0.005) m = - 0.005m

$$\frac{q}{A} = -k \cdot \frac{dT}{dx} = 4700 \ W/m^2$$

$$k = -\frac{q}{A} \cdot \frac{dT}{dx} = 4700 \times 0.005/35 = 0.67 \text{ W/m} \cdot \text{K}$$

Assumptions are that:

- Heat transfer is only through conduction
- The food composition or structure is maintained throughout the experiment (i.e. food is not decomposed by heating).

Conductive heat transfer in a Rectangular slab or plate

In practice, many food materials undergoing heat transfer have geometry that can be approximated as slabs or plate. By applying Fourier's law, it is possible to determine the temperature at any location inside a rectangular slab (Figure 1) under steady state condition.

Given that $q_x = -k \cdot A \cdot \frac{dT}{dx}$

The boundary conditions are:

$\text{T} = \text{T}_1 \ @ \quad x = x_1$

$\text{T} = \text{T}_2 \ @ \quad x = x_2$

Separating the variables,

$$\frac{q_x}{A} dx = -k dT$$

Integrating from x_1 to x (some interior location within the slab)

$$\int_{x_1}^{x} \frac{q_x}{A} dx = -\int_{T_1}^{T} k dT$$

$$\frac{q_x}{A}(x - x_1) = -k(T - T_1)$$

$$T = T_1 - \frac{q_x}{kA}(x - x_1)$$

$$q_x = -kA\frac{(T - T_1)}{(x - x_1)}$$

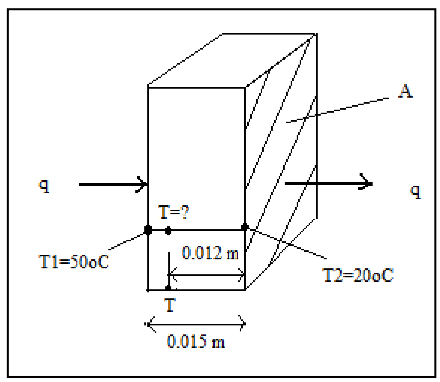
Thus, temperature (T) at any location x in the slab can be determined using:

$$T = T_1 - \frac{q_x(x - x_1)}{k \cdot A} \quad (2)$$

<u>Example 3</u>

Consider the metal plate in Example 1. Calculate the temperature at

i. T at 1.2 cm from 20°C surface



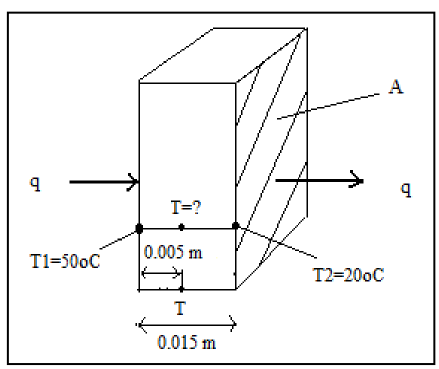
$$T = T_1 - \frac{q_x dx}{k \cdot A}$$

From T_1, dx = (0.015-0.012) = 0.003m

Therefore,

T= 50 – 34,000 (0.003)/17 = 49.4°C

ii. T at 1.0 cm from 50°C



$$T = T_1 - \frac{q_x dx}{k \cdot A}$$

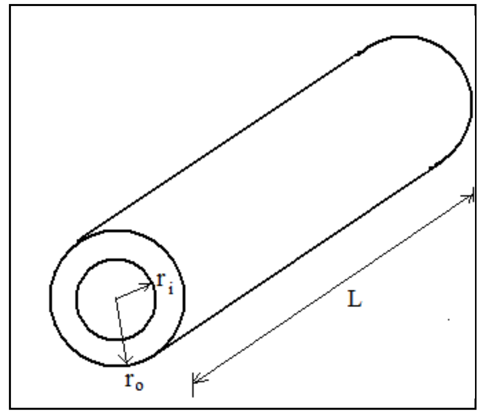
From T_1, dx = 0.01 m

Therefore,

T_1 = 50 – 34,000 (0.01)/17 = 30°C

Conductive heat transfer through a tubular pipe

Consider a long hollow cylinder of inner radius r_i and outer radius r_o and Length L. Let the inside wall surface be T_i, and outside surface temperature be T_o. Assume that thermal conductivity of the metal remains constant with temperature, the rate of heat transfer is obtained from Fourier law:



$$q_r = -k \cdot A \cdot \frac{dT}{dr}$$

A= 2πrL

Hence,

$$q_x = -k \cdot 2\pi r L \cdot \frac{dT}{dx} \qquad (3)$$

The boundary conditions are:

$$\mathrm{T} = \mathrm{T}_1 \quad @ \quad r = r_1$$

$$\mathrm{T} = \mathrm{T}_2 \quad @ \quad r = r_2$$

Rearranging equation (3) and integrating

$$\frac{\mathrm{q}}{2\pi \mathrm{L}} \int_{r_i}^{r_o} \frac{dr}{r} = -k \int_{T_i}^{T_o} dT \qquad (4)$$

$$\frac{\mathrm{q}}{2\pi \mathrm{L}} \ln r \Big|_{r_i}^{r_o} = -k \cdot T \Big|_{T_i}^{T_o}$$

Thus,

$$\mathrm{q} = \frac{2\pi L k (T_i - T_o)}{\ln(r_i / r_o)}$$

Or if $T_i > T_o$

$$\frac{\mathrm{q}}{2\pi \mathrm{L}} \ln r \Big|_{r_i}^{r_o} = -k \cdot T \Big|_{T_o}^{T_i}$$

Thus,

$$\mathrm{q} = \frac{2\pi L k (T_o - T_i)}{\ln(r_i / r_o)}$$