MTS 105
LECTURE NOTE :

SURDS

Definition 1: Any number which can be expressed as a quotient $\frac{m}{n}$ of two integers ($n \neq 0$), is called a rational number. Any real number which is not rational is called irrational. Irrational numbers which are in the form of roots are called **surds**. For example, $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$ and $3\sqrt{2}$ are irrational numbers while $\sqrt{16}, 3\sqrt{8}$ and $5\sqrt{32}$ can be expressed in rational form.

Definition 2: A general surd is an irrational number of the form $a\sqrt[n]{b}$, where a is a rational number and $\sqrt[n]{b}$ is an irrational number, while $\sqrt[n]{}$ is called a radical.

RULES FOR MANIPULATING SURDS

(i) $a\sqrt{b} + c\sqrt{b} = (a+c)\sqrt{b}$. This is the addition law of surds with the same radicals.

(ii) $a\sqrt{d} - c\sqrt{d} = (a-c)\sqrt{d}$. This is the subtraction law of surds with the same radicals.

(iii) $\sqrt{ab} = \sqrt{a}.\sqrt{b}$.

(iv) $(a\sqrt{b}).(c\sqrt{d}) = ac\sqrt{bd}$.

(v) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(vi) $(a\sqrt{b}) \div (c\sqrt{d}) = \frac{a}{c}\sqrt{\frac{b}{d}}$

(vii) $(\sqrt{a})^2 = a = \sqrt{a^2}$

(viii) $(\sqrt{a})^n = \sqrt{a^n}$

(ix) $\sqrt{a^{-m}} = \frac{1}{\sqrt{a^m}}$

(x) $\frac{1}{\sqrt{a^{-m}}} = \sqrt{a^m}$

Simplification of surds

Example; Simplify the following (i) $\sqrt{75}$ (ii) $\sqrt{80}$ (iii) $\sqrt{18}$ (iv) $\sqrt{60}$

Solution: Using rule 3

(i) $\sqrt{75} = \sqrt{25 \times 3} = \sqrt{25}.\sqrt{3} = 5\sqrt{3}$

(ii) $\sqrt{80} = \sqrt{16 \times 5} = \sqrt{16}.\sqrt{5} = 4\sqrt{5}$

(iii) $\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9}.\sqrt{2} = 3\sqrt{2}$

(iv) $\sqrt{60} = \sqrt{4 \times 15} = \sqrt{4}.\sqrt{15} = 2\sqrt{15}$

Addition and subtraction of surds

Example; Simplify the following (i) $\sqrt{50} - \sqrt{18} + \sqrt{32}$ (ii) $\sqrt{80} + \sqrt{20} - \sqrt{45}$ (iii) $\sqrt{28} + \sqrt{63}$

Solution: Using rule 1 and 2

(i) $\sqrt{50}-\sqrt{18}+\sqrt{32}$
$=\sqrt{25\times 2}-\sqrt{9\times 2}+\sqrt{16\times 2}$
$=5\sqrt{2}-3\sqrt{2}+4\sqrt{2}$
$=6\sqrt{2}$

(ii) $\sqrt{80}+\sqrt{20}-\sqrt{45}$
$=\sqrt{16\times 5}+\sqrt{4\times 5}-\sqrt{9\times 5}$
$=4\sqrt{5}+2\sqrt{5}-3\sqrt{5}$
$=3\sqrt{5}$

(iii) $\sqrt{28}+\sqrt{63}$
$=\sqrt{4\times 7}+\sqrt{9\times 7}$
$=2\sqrt{7}+3\sqrt{7}$
$=5\sqrt{7}$

Rationalization of surds

A surd of the form $\frac{\sqrt{3}}{2}$ cannot be simplified, but $\frac{2}{\sqrt{3}}$ can be written in a more convenient form. Then, we multiply the numerator and denominator of $\frac{2}{\sqrt{3}}$ by $\sqrt{3}$. Such that $\frac{2}{\sqrt{3}}=\frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$. This process is called rationalization.

Useful hints on rationalization of surds

(i) $\sqrt{a}.\sqrt{a}=a$
(ii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})=a-b$
(iii) $(x\sqrt{a}+y\sqrt{b})(x\sqrt{a}-y\sqrt{b})=x^2a-y^2b$
(iv) $(x+y\sqrt{b})(x-y\sqrt{b})=x^2-y^2b$
(v) the conjugate of $a+\sqrt{b}$ is $a-\sqrt{b}$

Example; (i) Rationalize $\frac{a}{n\sqrt{b}}$ (ii) if $\sqrt{3}=1.732$, find the value of $\frac{2}{\sqrt{3}}$ correct to 3 significant figure (iii) Express $\frac{8-3\sqrt{6}}{2\sqrt{3}+3\sqrt{2}}$ in the form $m\sqrt{3}+n\sqrt{2}$ where m and n are rational numbers

(iv) Express $\frac{2}{(3\sqrt{5}-4)^2}$ in the form $a+b\sqrt{c}$, where a and b are rational numbers

Solution:

(i) $$\frac{a}{n\sqrt{b}}=\frac{a\sqrt[n]{b^{n-1}}}{\sqrt[n]{b}\sqrt[n]{b^{n-1}}}=\frac{ab^{\frac{n-1}{n}}}{b^{\frac{1}{n}}.b^{\frac{n-1}{n}}}=\frac{a\sqrt[n]{b^{n-1}}}{b}$$

(ii) $\frac{2}{\sqrt{3}}=\frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{3}=\frac{2\times 1.732}{3}=1.16$ correct to 3 sig.figure.

(iii) $\frac{8-3\sqrt{6}}{2\sqrt{3}+3\sqrt{3}}=\frac{8-3\sqrt{6}}{2\sqrt{3}+3\sqrt{2}}\times\frac{2\sqrt{3}-3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}}=\frac{16\sqrt{3}-24\sqrt{2}-6\sqrt{18}+9\sqrt{12}}{12-18}$

$$=\frac{16\sqrt{3}-24\sqrt{2}-18\sqrt{2}+18\sqrt{3}}{-6}=\frac{34\sqrt{3}-42\sqrt{2}}{-6}=-\frac{17}{3}\sqrt{3}+7\sqrt{2}$$

$$\Rightarrow m=-\frac{17}{3} \quad and \quad n=7$$

(iv) $\frac{2}{(3\sqrt{5}-4)^2}=\frac{2}{45-24\sqrt{5}+16}=\frac{2}{61-24\sqrt{5}}=\frac{2(61+24\sqrt{5})}{(61-24\sqrt{5})(61+24\sqrt{5})}$

$$\frac{122}{841}+\frac{48}{841}\sqrt{5},\Rightarrow a=\frac{122}{841},b=\frac{48}{841},c=5$$

Equations involving surds

Example; (i) Solve the equation $\sqrt{(3x+1)}-\sqrt{(x+4)}=1$

(ii) simplify $\sqrt{5+2\sqrt{6}}$ (iii) Evaluate $\sqrt{9-4\sqrt{2}}$

Solution:

(i) $\sqrt{(3x+1)}-\sqrt{(x+4)}=1\equiv\sqrt{(3x+1)}=1+\sqrt{(x+4)}$ (1)

squaring both sides of (1), we have

$$3x+1=[1+\sqrt{(x+4)}]^2\Rightarrow 3x+1=1+2\sqrt{(x+4)}+x+4$$

$$\Rightarrow 3x+1=x+5+2\sqrt{x+4}\Rightarrow 2x-4=2\sqrt{(x+4)} \qquad (2)$$

$$\Rightarrow x-2=\sqrt{(x+4)}$$

squaring both sides of (2) again yields

$$(x-2)^2=x+4\Rightarrow x^2-5x=0\Rightarrow x=0 \quad or \quad x=5$$

$$if \quad x=0,\Rightarrow\sqrt{(3x+1)}-\sqrt{(x+4)}=-1(not\,solution)$$

$$if \quad x=5,\Rightarrow\sqrt{(3x+1)}-\sqrt{(x+4)}=1$$

$$\Rightarrow x=5 \quad is \quad the \quad solution$$

(ii) Let $\sqrt{5+2\sqrt{6}}=\sqrt{x}+\sqrt{y}$ (1)

Squaring both sides of (1), we obtain

$$5+2\sqrt{6}=x+y+2\sqrt{xy}$$

$$\Rightarrow 5=x+y, \qquad 6=xy$$

By inspection, $x=3, \quad y=2$

$\Rightarrow \sqrt{5+2\sqrt{6}} = \sqrt{3}+\sqrt{2}$

(iii) Let $\sqrt{9-4} = \sqrt{2 =}\sqrt{x-\sqrt{y}}$ (1)

The conjugate of (1) is

$\sqrt{9+4}\sqrt{2} = \sqrt{x}+\sqrt{y}$ (2)

Squaring both sides of (1), we have:

$9-4\sqrt{2 =} - 2\sqrt{xy}$

$\Rightarrow x+y=9$ (3)

Multiplying (1) and (2), we obtain

$\sqrt{9x9-16.2} = x-y=7$ (4)

From (3) and (4)

$\Rightarrow x+y=9$

$x-y=7$

$\therefore 2x=16$

$x=8, y=1$

EXERCISES

(1) (i) $\sqrt{405}$ (ii) $\sqrt{98}$ (iii) $\sqrt{27 =} - \sqrt{12}$ (iv) $\left(\sqrt{7}-\sqrt{5}\right)^2$ (v) $\dfrac{1}{\sqrt{3-1}}$ (vi) $\dfrac{3}{\sqrt{7-2}}$

(vii) $\dfrac{3}{\sqrt{7-2}}$ (vii) $\dfrac{\sqrt{3-1}}{\sqrt{3+1}}$ (viii) $\dfrac{2\sqrt{2+3}}{2\sqrt{2-1}}$

(2) If $a=2+\sqrt{3}$, Find the value of $a-\dfrac{1}{a}$

(3) Given $a\dfrac{1}{2-\sqrt{3}}$, $b\dfrac{1}{2+\sqrt{3}}$, find a^2+b^2

(4) Find the positive square roots of the following :

(i) $19+6\sqrt{2}$ (ii) $43+12\sqrt{7}$

(5) If $x=\dfrac{1}{2}\left(1-\sqrt{5}\right)$, express $4x^3-3x$ in its simplest form.

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INDICES

Definition 1: The product of a number with itself called the second power of the number, while the number, while its triple product is called third power of the number and its m factors product is called mth power of the number e.g. $axa = a^2$, $axaxa = a^3$, $axax...xm = a^m$

Definition 2: The number which expresses the power is called the index or the exponent of the power of a number e.g

The index of $a^2 = 2$
The index of $a^3 = 3$
The index of $a^m = m$

RULES OR LAWS OF INDICES

Given two positive integers m, n such that $m < n$.

(1) $a^m x a^n = a^{m+n}$

Since $a^m x a^n = (axax...xm) x (axax...xn)$

$[axax...x(m+n)] = a^{m+n}$

(2)

$= axaxax...(m-n)$

$= a^{m-n}$

LOGARITHMIC EQUATIONS

Example 1: Solve the equations

(i) $3x^2 = 9^{x+4}$

$\Rightarrow 3x^2 = (3^2)^{x+4}$

$3x^2 = 3^{2(x+4)}$

$\Rightarrow x^2 = 2(x+4)$

$\Rightarrow x^2 = 2n - 8 = 0$

$(x-4)(x+2) = 0$

$x - 4$ or $x = 2$

Example 2: Solve the equations

(ii) $3^{3x+1} = 5^{x+1}$

Taking the log; of both sides

$\Rightarrow \log_{10}(2^{3x+1}) \log_{10} 5^{x+1}$

$\Rightarrow 3x + 1Log_{10} 2 = x + 1Log_{10} 5$

$$\Rightarrow \left(3Log_{10} - Log_{10}5 \right)x = Log5 - Log_{10}2$$

$$\Rightarrow \left(3Log_{10}8 - Log_{10}5 \right)x = Log5 - Log_{10}2$$
$$\Rightarrow \left(Log_{10}8 - Log_{10}5 \right)x = Log5 - Log_{10}2$$
$$\Rightarrow x = \frac{Log_{10}5 - Log_{10}2}{Log_{10}8 - Log_{10}5} = \frac{Log_{10}\frac{5}{2}}{Log_{10}\frac{8}{5}} - Log_{10}2$$

$$\Rightarrow x = \frac{0.3979}{0.2041} = 1.95$$

(3) $(a^m)^n = a^{mn}$

$(a^m)^n = a^m x a^m x \ldots xn$

$(axax\ldots xm)x(axax\ldots xm)\ldots n$ times.

$axax\ldots xmn = a^{mn}$

(4) $\left(a\frac{1}{n} \right)^v = a$

$\left(a\frac{1}{n} \right) a\frac{1}{n} xa\frac{1}{n} x\ldots n$

$a\frac{1}{n} + \frac{1}{n} + \frac{1}{n} + \ldots a$

Similarly, $a\frac{1}{n} = n\sqrt{a}$

$\left(a\frac{m}{n} \right)^n = a^m$

$a\frac{m}{n} = n\sqrt{a^m} = \left(n\sqrt{a^m} = \left(n\sqrt{a} \right)^m \right.$

(5) $a^0 = 1$, If $m = n$

$$a^{-n} = \frac{1}{a^n}$$

Examples: Evaluate (i) $(81)^{\frac{3}{4}}$ (ii) $(16)^{\frac{-5}{4}}$

Solution

(i) $(81)^{\frac{3}{4}}$ $= (81^3 = 4\sqrt{531441 = (3^4)^{\frac{3}{4}} = 3^3 = 27}$

(ii) $(16)^{\frac{-5}{4}} = \dfrac{1}{(2^4)\frac{5}{4}} = \dfrac{1}{2} = \dfrac{1}{32}$

Exercises

(1) Show that $\sqrt{x} - \sqrt{a} = \dfrac{x-a}{\sqrt{x}+\sqrt{a}}$

(2) Evaluate (Godman).

LOGARITHMS

Definition: The logarithm of a tve no N to the base a is defined as the power of a which is equal to N, such that if

$$a^x = N$$
$$x = Log_a N$$

Since $a^1 = a \delta a^0 = 1$

$\Rightarrow Log_a a = 1$ and $Log_a 1 = 0$

LAWS OF LOGARITHMS

(1) $\log_a (AB) = Log_a A + Log_a B$

(2) $\log_a \dfrac{A}{B} = Log_a A - Log_a B$

(3) $\log_a (A^B) = BLog_a A$

Example: Evaluate:

(i) $Log_3 9$

(ii) $Log_4 63$

(iii) $3^3 9 \Rightarrow Log_3 9 = 2$

(iv) $4^3 = 64, \Rightarrow Log_4 64 = 3$

Example: Use the table to evaluate:

(i) $Log_3 16 = \dfrac{Log_{10} 16}{Log_{10} 3} = \dfrac{1.2041}{0.4771} = 2.524$

Since from the transformation rule

$Log_9 N = Log_{10} b Log_6 N$

If $y = Log_b N, N = b^y$

$\Rightarrow Log_a N = Log(b^y) = y = b^y$

$Log_a N = Log_a b Log_b N$

If we put $N = a$ in (*)

$\Rightarrow Log_a a = Log_a b Log_b a = 1$

$$\Rightarrow Log_a b = \frac{1}{Log_a b} \qquad (**)$$

Another form of (*) is $Log_a N = \dfrac{Log_b N}{Log_b a}$

Example: Show that:

$$Log_a (x^2 - x^2) = 2 + Log_a (1 - \frac{x^2}{a^2})$$

Solution:

$$Log_a (a^2 - x^2) = Log_a [\ a^2 + (1 - \frac{x^2}{a^2})\ ]$$

$$= Log_a a^2 + Log_a (1 - \frac{x^2}{a^2})\ ]$$

$$= 2 + Log_a (1 - \frac{x^2}{a^2})\ ]$$

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SET THEORY

Definition: A set is a collection of objects or things that is well defined.

Here are some examples of sets:

1. A collection of students in form one
2. Letters of the alphabet
3. The numbers 2, 3, 5, 7, and 11
4. A collection of all positive numbers
5. The content of a lady's purse

The concept of set is very important because set is now used as the official mathematical language. A good knowledge of the concept of set is, therefore, necessary if mathematics is to be meaningful to its users.

Notation

A set is usually denoted by capital letters; while the objects comprising the set are written with small letters. These objects are called members or elements of a set.

For example set A has members $a, b, c, d.$

Convention

The listing of a set A as $a, b, c, d,$ as seen above is not an acceptable mathematical specification of a set. The correct representation of a set that is listed is to write the elements, separated by commas and enclosed between braces or curly brackets.

e.g., set $A = \{\ a, b, c, d.\ \}.$

The statement b is an element or member of set A or b belongs to a' is written in the manner $b \in A.$ The contrary statement that b does not belong to A is written as: $b \notin A.$

There are two ways of specifying a set. One way is by listing the elements in the set, such as:

$A = \{\ a, b, c, d.\ \}.$

A second way of specifying a set is by stating the rule or property which characterizes the set.

For example, $B = \{\ x / 2 < x < 5.\ \}$or $B = \{\ x / 2 < x < 5.\ \}.$ Notice, the stroke/or colon: can be used interchangeably, with each as 'such that'. The representation, $B = \{\ x / 2 < x < 5.\ \}$ is read as follows: B is a set consider of elements $x,$ such that 2 is less than x and x is less than 5.

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If a set is specified by listing its elements, we call it the tabular form of a set; and if it is specified by stating its property, such as $C = \{ x / x \text{ is odd} \}$, then it is called the set builder form.

Finite and Infinite Sets

A finite set is one whose members are countable: for example, the set of students in Form 1. Other examples are:

(i) the contents of a lady's hand-bag;
(ii) whole numbers lying between 1 and 10;
(iii) members of a football team.

The finite set is itself in exhaustive; readers can give other examples of a finite set.

An infinite set is one whose elements are uncountable, as they are infinitely numerous. Here are a few examples of the infinite sets;

(i) Real numbers.
(ii) Rational numbers
(iii) Positive even numbers
(iv) Complex numbers

The main distinction between a finite set and that a finite set has a definite beginning and a definite end, while the infinite set may have a beginning and no end or vice versa or may not have both beginning and end.

For example, we specify the set of positive even numbers, as follows:

$P = \{ 2,4,6,... \}$ or
$P = \{ x : x > 2, x \text{ is even} \}$

The set of real whole numbers which end with the number 3 is written as follows:

SUBSETS

Suppose $P = \{ a,b,c,d,e,f \}$ and $Q = \{ c,d,e. \}$, then we say Q is contained in P, and we use symbol $'\subset'$ to denote the statement 'is contained in', or 'is subset of'. Thus $Q \subset P$, is ready as $'Q$ is contained in P'. More aptly put, Q is contained in P if there is an x, such that $x \in Q$ implies $x \in Q$. The statement Q is contained in P can be put in reverse order as $'P$ contains Q' and we write $P \supset Q$. However, this form is not very popular. If Q is not a subset of set $R = \{ 3,4,a \}$, then we write $Q \not\subset R$. It should be noted that unless every member or Q is also a member of P, then can we say Q is subset of P.

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EQUITY OF SETS

Two sets of X and Y are equal if and only if $X \subset Y$ and $Y \subset X$. Suppose $X = \{ 1,2,3 \}$ and $Y = \{ 3,1,2 \}$ and $X = Y$. Note that the rearrangement of the elements if a set does not alter the set.

TYPES OF SETS

Null or Empty Sets

Null means void, therefore, a null set is an empty set, or a set that has no members. The null set is denoted by the symbol $\{ \}$. Note that $\{0\}$ cannot be classified as a null set, because it has an element, zero.

Singleton

Any set which has only one member is called a singleton. e.g., (a) is a singleton.

The Universal Set

Set is a subset of a larger set is called the universal set or empty, the **Universe of Discourse.**

Thus, in any given context, the total collection of elements under discussion is called the Universal set.

The symbol U or E is often used to denote a universal set. For example, if we toss a die, once, we expect to have either $1,2,3,4,5,$ or $6,$ as an end result. If there are no other expected results different from this numbers, then we say, for this particular experiment, the universal set is $\{ 1,2,3,4,5,6 \}$. Thus a universal set is the total population under discussion.

Proper Subsets

If P is a subset of Q and if there is at least one member of Q which is not a member of P, then P is a proper subset of Q and we write $P \subset Q$.

Consider the set $A = \{ 1,2,3, \}$. The following sets
$\{ 1,2,3, \}, \{ 1,2, \}, \{ 1,,3, \}, \{ 2,3, \}, \{ 1, \}\{ 2, \}, \{ 3, \}, \{ \}$ are subsets of A. The set $\{ 1,2,3 \}$ is not a proper subset of A; whereas all others including $\{ \}$ are proper subsets of A. Thus $\{ 1,2,3 \} \not\subset \{ 1,2,3 \}$, but $\{ 1,2,3 \}, \{ 1,2,3 \} \subseteq \{ 1,2,3 \}$.

Power Set

The collection of all the subsets of any set S is called the power set of S . If a set has n members, where n is finite, then the total number of subsets of S is 2^n. Occasionally we denote the power set S by 2^S.

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For example: Let $A = \{a, b, c\}$. The subsets of A are $\{a,b,c\}, \{a,b\}$, $\{a,c\}, \{b,c\}$, $\{a\}, \{b\}, \{c\}$ and $\{ \ \}$.
The power set of A written $p(A) = 2^3$ subsets; as seen above.

Example: Find the power set 2^S of the sets

(a) $S = \{ \ 3,4)$
(b) $S = \{ \ a, \{ \ ,1,2 \ \} \ \}$

Solution:

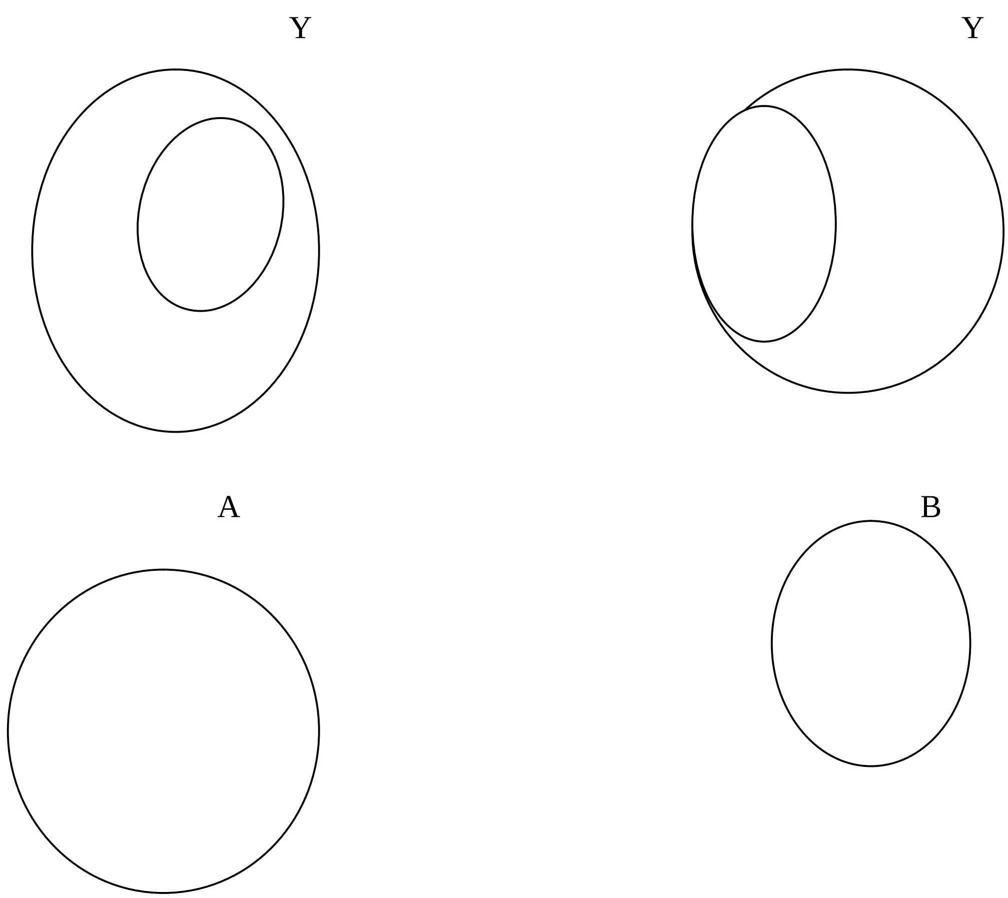
(a) $S = \{ \ 3,4 \ \}, \{ \ 3 \ \}, \{ \ 4 \ \}, \{ \quad \},$
(b) $2^P = \{ \ a, \{ \ ,1,2 \ \} \ \}, \{ \ a \ \}, \{ \ 1,2 \ \}, \{ \quad \}$

In this example, (b) contains only two elements a and $\{ \ 1,2 \ \}$

Venn – Euler Diagrams

The theory of set can be better understood if we make use of the Venn-Euler diagrams. The Venn-euler diagram is an instructive illustration which depicts relationship between sets.

Suppose $X \subset Y$ and $X \not\subset Y$, we can represent this statement in a Venn-Euler diagram as follows:



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Set Operations

In set, we use the symbols $\cup$ read 'unions and $\cap$ read 'intersection' as operations. These operations are similar but not exactly the same as the operations in arithmetic. At the end of this chapter, a reader of this topic should be able to identity areas of analogy between the operation in arithmetic and those of a set.

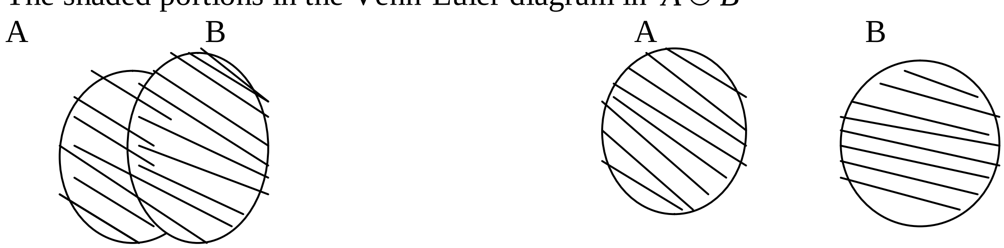
Union of Sets

Definition

The union of sets A and B is the set of all elements which belongs to A or B or to both A and B. This is usually written as $A \cup B$, and read $'A$ union B'.

In set language, we define $A \cup B$ as:
$A \cup B = \{ x : x \in A \text{ or } \in B \}$.

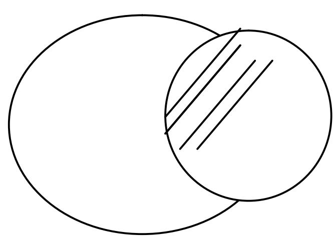
The shaded portions in the Venn-Euler diagram in $A \cup B$



The Intersection of Sets

The intersection of sets $A and B$ is the set of elements which belong to both $A and B$. Simply, $'A$ intersection B' written $A \cap B$ consists of elements which are common to both $A and B$.

The Venn-Euler diagram which represent $A \cap B$ is shaded portion.



In set language

$A \cap B = \{ x : x \in A and x \in B$

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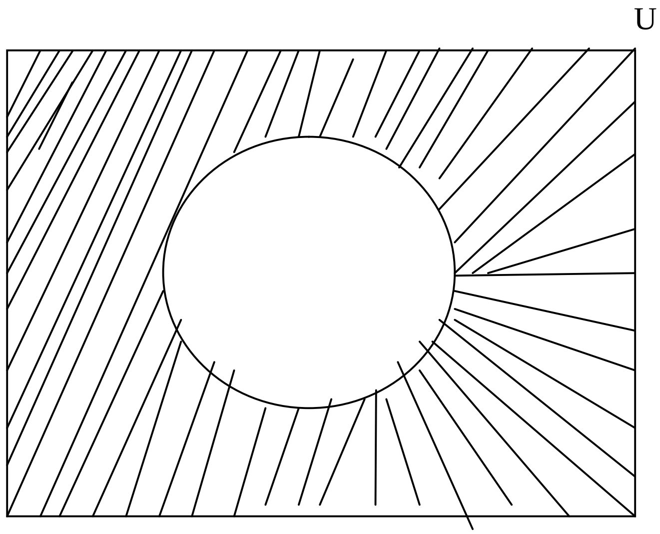
Complement of Sets

The complement of a set x is the set of elements which do not belong to x, but belong to the universal set. The complement of a set x is usually represented by x' or $x^{c.}$

The complement of x' or x^c.

The complement of x is represented in the Venn-Euler diagram



In set language, $A^c = x : x \in U, x \notin A$

The Algebra of Sets

The operations of union $\cup$ are loosely analogous to those of addition and multiplication in number algebra. By this token we can apply the laws of algebra conveniently to sets without loss of generality.

The Closure property

If X and Y are sets which are subsets of the universal set U then the following hold:

$X \cup Y \subset U$ and $X \cap Y \subset U$.

The analogy in number algebra; using those operations of $+$ and x are $2+3=5 \in R$ and $2x3 = 6 \in R$; where R is the real number system. If the addition or multiplication of 2 and 3 gives some number that cannot be found in the real number system R, we say the operation of $+$ or x is not closed.

Similarly in set theory, the operations of union and intersection are closed.

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The Commutative Law

$X \cup Y = Y \cup U \quad X \cap Y = Y \cap X$. Parallel examples in arithmetic are $2+3=3+2$ and $2X3=3X2$.

Thus any two sets are commutative with respect to $\cup$ and $\cap$.

The Associative Law

$X \cup (Y \cup Z) = (X \cup Y) UZ$ and
$X \cap (Y \cap Z) = (X \cup Y) \cap Z$

Again, sets obey the associative law.

The Identity

In every day arithmetic, $0+1=1+0=1$ and $3X1=1X3=3$, are two correct solutions. The zero, in the first case is called the additive identity; while 1 in the second case is called the multiplicative identity.

By a similar analogy , every set has quantities $\{\ \}$ and U with the property that:

(i) $X \cup \{\ \} = \{\ \} \cup X = X$
(ii) $X \cap U = U \cap X = X$

Thus, $\{\ \}$ is the identify with respect to union $\cup$ and U is the identity with respect to intersection $\cap$.

Inverse

In the set of real number R,

$a + (-a) = (-a) + a = 0$ and $aXa^1 = a^1 Xa = 1$. This a number x operated on its inverse gives identity. i.e., x Inverse $=$ identity.

Similarly in set theory, every set has an inverse with respect to the operations of $\cup$ and $\cap$

(i) $X \cup X' = X'U = U$ and
$X \cup U = U \cup X = U$

(iii) $X \cap X' = X' \cap X = \{\ \}$ and
$X \cap \{\ \} = \{\ \} X' X = \{\ \}$

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The distributive Law

$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$ and
$X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$
The operation of union is distributive over the operation of intersection and vice versa.

The Laws of complementation

(i) $X \cup X' = \cup$ and
(ii) $(X')= X$
(iii) $(X \cup Y)' = X \cap Y'$
(iv) $X \cap Y)' = X' \cup Y'$
(ii) – (iv) are called de Morgan’s Laws