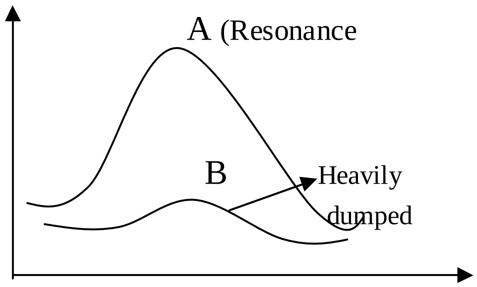
The amplitude of vibration and hence, the energy transferred into the vibrating system is found to depend on the difference between f and f_0, its maximum when the frequency of the external force is equal to the national frequency of the system.

i.e., $f = f_0$,
the amplitude can become large when the driving brief.
For lightly damped F, near the natural frequently ,
$f \approx f_0$. When the damping is small, the increase in amplitude near $f = f_0$ is very large. This effect is Known as **Resonance.** The natural vibrating frequency f_0 of a system is called its resonant frequency, f_0,frequency.



Assignment 4

1. A spring stretches 0.150m when a 0.300kg mass is hung from it. The spring is then stretched an additional 0.100m from its equilibrium point and released.

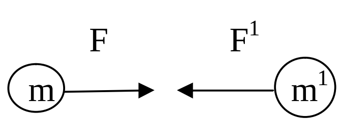
   Determine (a) the spring constant k (b) the amplitude of the oscillation A (c) the maximum velocity V_0 (d) the velocity v when the mass is 0.050m from equilibrium and (e) the maximum acceleration of the mass.

2. For a S.H.O. determine, (a) the total energy (E), the kinetic and potential energies of half amplitude: $\left(x = \pm \frac{A}{2}\right)$.

4.0 Gravitation

Newton's study on planetary motion has led to inferring a formular for gravitational force between two masses. This formula is termed the law of universal gravitation (law of nature). It states that for two uniform spheres of two objects of any shape that are so small compared with their separation, that, they may be considered as point particles, the law has a simple form. If two spheres or particles have gravitational masses m &m1 and their centres are separated by distance r, then the forces between the two spheres have a magnitude.

$F = \frac{Gmm^1}{r^2}$



G = Gravitational constant = 6.67×10^{-11} NM ^{2}kg $^{-2}$

Since the magnitude of the gravitational force varies as $\frac{1}{r^2}$,

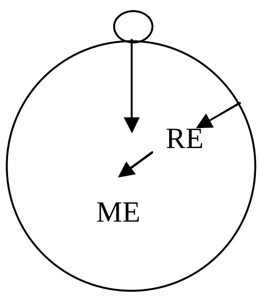
Then the law is called **Inverse Square Law**

Every particle in the universe attracts every other particle with a force that is proportional to the square of the distance between them. This force acts along the path of the two particles.

4.1 Weight

Weight of an object is the gravitational force it experiences. For an object on the surface of the Earth, its force is mainly due to the earth's attraction.

Consider an object with gravitational mass m at the surface of the earth, subjected to a gravitational force, **F,** by law of universal gravitation.



RE = Radius of the earth = 6400km

$$\therefore F = \frac{G_m M_E}{R^2{}_E}$$

Since F = m_a (Newton's gud law)

$$m_a = \frac{G_m M_E}{R^2{}_E}$$

a = g

$$m_g = \frac{G_m M_E}{r^2{}_E}$$

$$g = \frac{G\ M_E}{R^2{}_E}$$

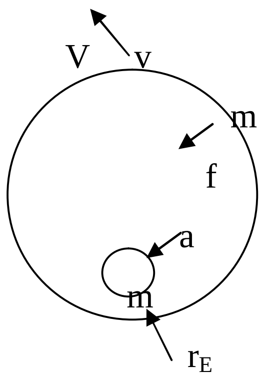
i.e, gravitational acceleration is the same for all objects.

<u>Satellites & Weightlessness</u>

A satellite is put in an orbit by accelerating it to a sufficiently high tangential speed with the use of rocket. For instance in circular motion, satellites are usually put into circular orbits because they require at least take-off speed. Therefore, for satellites that move in orbit, it acceleration is, $\frac{v^2}{r}$, Hence recall that F = $\frac{G_m M_E}{r^2{}_E}$ and since

$a = \frac{v^2}{r}$ and F = $m_a = \frac{mv^2}{r}$

$\therefore \frac{G_m M_E}{r^2{}_E} = \frac{mv^2}{r}$



m = mass of satellite

r = sum of the Earth's radius

r_E plus the satellite's height h

above the Earth: r = r_E th

V_v = velocity of the orbit

$$V_{orb} \quad \sqrt{\frac{GM_E}{r}}$$

Note: The mass of the satellite does not appear and the orbital speed decreases as the radius of the orbit increases.

Since velocity = $\frac{displacement}{Time}$ and displacement = 2πr, T = $\frac{2\pi r}{V_{orb}}$

Hence the period of the orbit is:

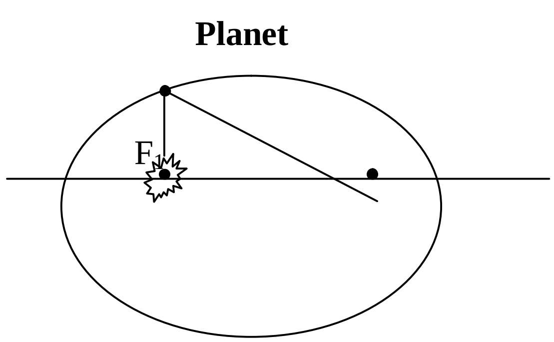
T = $\frac{2\pi r}{V_{orb}} = \frac{2\pi r}{\sqrt{GM}} \ x \ \sqrt{r^3}$

Square both sides

$T^2 = \frac{4\pi^2}{GM} \ x \ r^3 = kr^3$, This is called the **Kepler's third law,** which states that the square of the period of the orbit is proportional to the cube of the radius of the orbit.

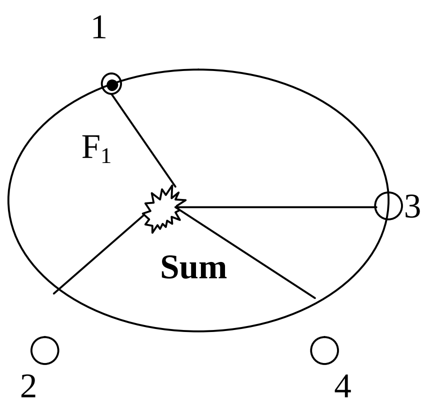
Summary of Keepler's Laws (Laws of Planetary Motion)

First Law: The path of each planet about the sum is an ellipse with the sun at one focus.



Sum F_2

Second Law: Each planet moves so that an imaginary line drawn from the sin to the planet sweeps out equal areas in equal periods of time



5.0 Statics and Hydrostatics

Statics: Study of forces in equilibrium

5.1 Mass, Forces and Weight

Just give definitions and then read-up:

5.2 Forces in Equilibrium

A single force cannot exist alone and is unbalanced. For equilibrium, it must be balanced by an equal and opposite force acting along the same straight line. Forces may be said to exist in pairs, however, a single force may also be balanced by any number of other forces.

Conditions for Equilibrium:

(1) Find a body to be at rest, the sum of the forces acting on it must add up to zero. Hence, if the forces on the object act in a plane, a condition for equilibrium is that $\sum F_k = 0$, $\sum F_y = 0$, and if it acts in 3-dimension, $\sum F_z = 0$,

(2) The sum of the torgues acting on a body must be zero

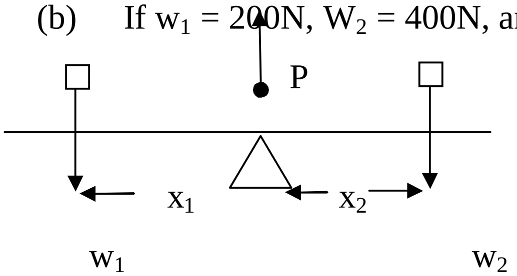
$$\sum \tau = 0$$

These two conditions ensure that a rigid body will be in both translationl and rotational equilibrium.

Example: Two weights w_1 and w_2 are balanced on a board pivoted about its centre: (

(a) What is the ratio of their distance $\frac{x_2}{x_1}$ from the pivot?

(b) If w_1 = 200N, W_2 = 400N, and x = 1m



X_1 = 1m, what is x_2?

(assume the board to be weightless)

Solution: First law of equilibrium condition, N force exerted by the support must balance their weights such that the net force is zero.

i.e. N = $w_1 + w_2$ ($N - w_1 - w_2 = 0$)

so, tongue about each weight: $\tau_1 = x_1\, w_1$

$$\tau_2 = x_2\, w_1$$

Hence $\tau = \tau_1 + \tau_2 = 0$

$x_1\, w_1 - x_2\, w_2 = 0$

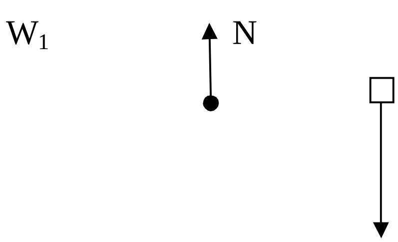
$x_1\, w_1 - x_2\, w_1$

$\frac{x_2}{x_1} = \frac{w_1}{w_2}$

(b) If w_1 = 200N, w_2 = 400N, $^0x_1$ = 5.0m

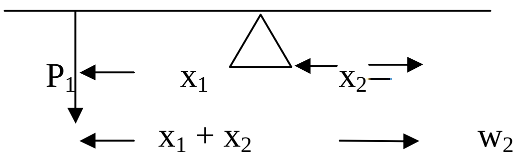
$$x_2 = x_1 \frac{w_1}{w_2} = \frac{200}{400} = 0.5\text{m}$$

Example 2: Again, find $\frac{w_1}{w_2}$ when the pivot is at the centre, i.e, torgue about point P, where w_1 is placed.



Torgue about (P_1), (N and w_2)

$x,\ N - (x_1 + x_2)\, w_2$



Smile, sum of torgues must be zero

$$(x_1 + x_2)\, w_2 + x_1 N = 0$$

Again, the sum of forces must be added up to zero

$N - w_1 - w_2 = 0$

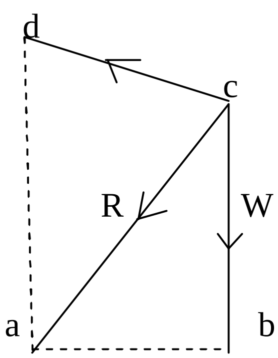
$N = w_1 + w_2$

∴ Putting N,

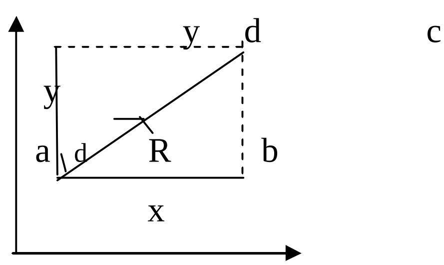
- $(x_1 + x_2)\, w_2 + (w_1 + w_2) x_1 = 0$
- $(x_1 + x_2)\, w_2 + (w_1 + w_2) x_1 = 0$
- $w_1 \; x_1 = x_2 \; w_2$
- $\frac{x_2}{x_1} = \frac{w_1}{w_2}$

Comment: No matter whereever P is, same answer is obtained.

5.3 Resolution of forces



Forces ab and ad can be replaced completely by a single force Ac i.e. to replace a single force by two other forces in any convenient direction. These two forces are known as the Components of the single force.

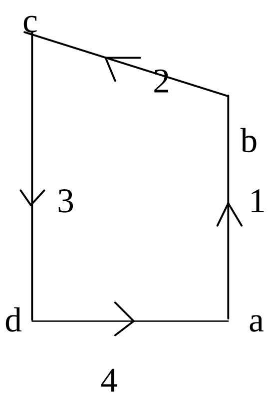


ab = ac cosθ i.e. x = R cos θ and d = ac sinθ i.e. Y = RSin θ

Polygon of Forces

If more than three forces act at the same point and are in equilibrium, they may be represented in magnitude, sense and direction by the sides of a polygon "taken in order"

1,2,3,4 are represented by ab, bc, cd and da



Hyrostatics (Fluid at Rest)

Preamble: Matter consists of 3 states: Solid, Liquid and Gases.

Fluid: Liquid and gases (has definite volume, but no define shape while gas has neither, definite, shape or volume.

Pressure: Study of fluid mechanics involves density of a substance (defined as mass per unit volume).

If **F** is the magnitude of the normal force on the paston and **A** is the surface area of the piston, then the pressure, **P** of the fluid at the level to which the device has been submerged, is defined as the ratio of force to area.

Pressure = $\frac{Force}{Area}$

Suppose the normal force exerted by the fluid is **F** over a surface element of area δA, the pressure at that point is:

$P = \frac{lin}{\delta A \to 0} \frac{F}{\delta A} = \frac{dF}{dA}$. Unit is N/m^2 (Pascal P_a)

Transmission of Fluid Pressure

Pressure increases linearly with depth. Consider a liquid of density **P** at rest and open to the atmosphere as in the figure below. A sample of liquid in a cylinder of cross-sectional area, A, extending from the surface of the liquid to a depth **h**, pressure exerted by the fluid on the bottom face is **P** and on the tip is $\mathbf{P_o}$, hence, upward force is PA and downward force exerted is $\mathbf{P_oA}$.

Mass of liquid in the cylinder is m = Pv = PAh.

Weight w of the liquid in the cylinder is W = mg = Pvg = for the cylinder to be in equilibrium, upward force must be greater than the downward force.

$PA – P_oA = PghA – A (P – P2) = PghA$

$P = P_o + Pgh$

P_o = l atm pressure ≈

$1.01 \times 10^5 P_a$

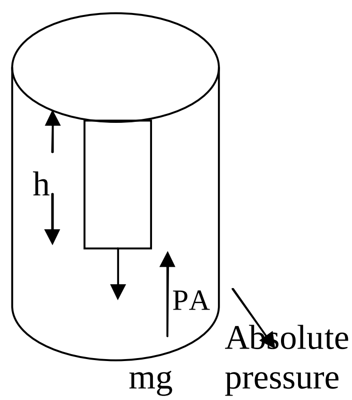


fig.

Pascal's Law

A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the liquid and to the wall of the container i.e. pressure at every point in a liquid is the same.

Fluid Dynamics (Fluids in Motion)

This is the study of properties of a fluid as a function of time. Fluid in motion in characterized in two main types: Steady or Laminar and Non-Steady or Turbulent.

Steady or Laminar: If each particle of the fluid follows a smooth path such that different particle never cross each other. In this case, the velocity of the fluid at any point remains constant in lime.

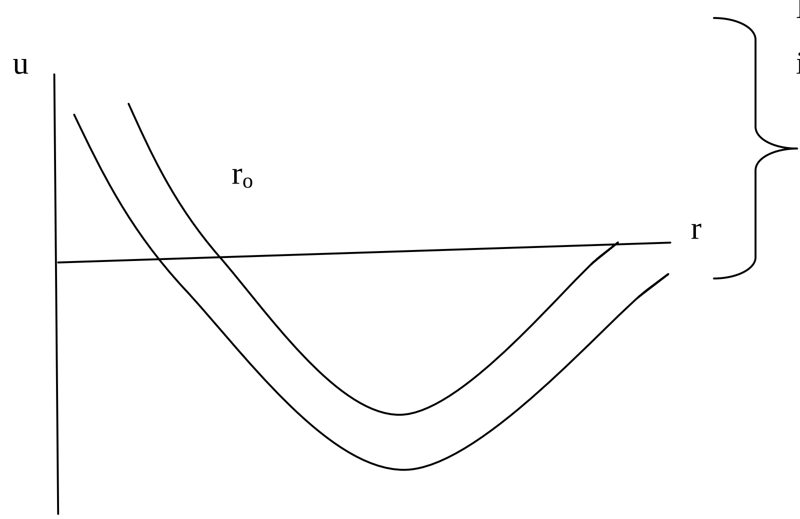
Non Steady or Turbulent: This is an irregular flow characterized by whirl pool–like region i.e. at certain critical speed, the fluid flow becomes non-steady.

Viscosity: Degree of internal friction in the fluid, viscos force is associated with the resistance of two adjacent layers of the fluid to move relative to each other.

Elasticity

Elasticity Properties of Solids

Pre-ambles: Kinetic theory of gasses has shown that matter consists of molecules, which behaves like free particles in gases. For solids, the molecules have small distance and so, exert significant forces on one another. The relationship between potential energy $U_{(r)}$ and $F_{(r)}$ is illustrated in the graph below:



Energy dependence of intermolecular distance.

Fig (2.0)

Note: That the molecules are normally at freed position (distance r_o from one another) and their forces in any molecule is zero; and potential energy (p.e) is minimum.

$$\frac{du_{(r)}}{dr} = 0$$

The response of a material to a given type of deforming force as explained in the above graph describes the principle of elasticity- the ability of a material to return back to its original shape and size.

Stress: This is the external force per unit cross sectional area acting on an object

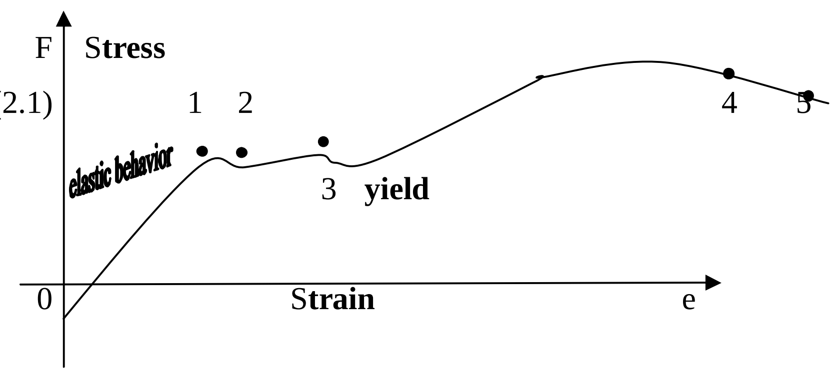
$$\text{Stress} = \frac{\text{Force}}{\text{Area}}, \text{ Unit is } Nm^{-2}$$

Strain: - This is the ratio of the change in original size or shape to the original size or shape.

$$\text{Strain} = \frac{\Delta R}{h}$$

Illustration: - A piece of wire stretched by an increasing force obeys Hook's law i.e. extension is proportional to force strain is proportional to stress.

Fig (2.1)



0 – 2, wire return to its original length after the load had been removed. **2,** is known as elastic limit. After this print, it is brittle. **3,** is the yield print beyond this print, extension is rapid, and this known as plasticity. At **4**, material is under maximum force or stress that can be sustained. At **5**, it is the breaking point.

Since strain is proportional to stress, the constant of protionality is called **elastic modulus**