

**MCE 308**

**APPLIED THERMODYNAMICS II**

**(3 UNITS)**

**PREPARED BY**

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# APPLIED THERMODYNAMICS

## 1.0 GASEOUS MIXTURES

### 1.1 DEFINITION

#### 1.1.1 Pure Substance

A pure substance is defined as a substance having a constant and uniform chemical composition.

#### 1.1.2 Homogeneous Mixture of Gases

Any mixture of gases in which the constituents do not react chemically with one another and they are in a fixed proportion by weight is referred to as homogeneous mixture of gases and is regarded as a single substance. The properties of such a mixture can be determined experimentally just as for a single substance, and they can be tabulated or related algebraically in the same way. Therefore, the composition of air is assumed invariable for most purposes and air is usually treated as a single substance.

#### 1.1.3 Mixture of Gases

Mixture of gases is generally imagined to be separated into its constituents in such a way that each occupies a volume equal to that of the mixture and each is at the same temperature as the mixture.

### 1.2 EMPIRICAL LAW FOR MIXTURES OF GASES

Consider a closed vessel of volume  $V$  at temperature  $T$ , which contains a mixture of perfect gases at a known pressure. If some of the mixture were removed, then the pressure would be less than the initial value. If the gas removed was the full amount of one of the

constituents then the reduction in pressure would be equal to the contribution of that constituent to the initial total pressure. This is not only applicable to pressure but also to internal energy, enthalpy and entropy.

### 1.2.1 Dalton's Law

In a mixture of gases, each constituent contributes to the total pressure by an amount which is known as the partial pressure of the constituent. The relationship between the partial pressures of the constituents is expressed by Dalton's law, as follows:

*The pressure of a mixture of gases is equal to the sum of the pressures of the individual constituents when each occupies a volume equal to that of the mixture at the temperature of the mixture.*

Dalton's law is based on experiment and is found to be obeyed more accurately by gas mixtures at low pressures. By the conservation of mass:

$$m = m_A + m_B + m_C + \dots\dots\dots \text{ or } m = \sum m_i \quad (1.1)$$

By Dalton's law

$$P = P_A + P_B + P_C + \dots\dots\dots \text{ or } P = \sum P_i \quad (1.2)$$

where  $m_i$  = mass of a constituent; and  $P_i$  = partial pressure of a constituent.

### 1.2.2 GIBBS-DALTON LAW

Dalton's law was reformulated by Gibbs to include a second statement on the properties of mixtures. The combined statement is known as the Gibbs-Dalton law, and is stated as follows:

*The internal energy, enthalpy and entropy of a mixture of gases are respectively equal to the sums of the internal energies, enthalpies and entropies of the individual constituents when*

each occupies a volume equal to that of the mixture at the temperature of the mixture. This statement leads to the following equations.

$$mu = m_A u_A + m_B u_B + m_C u_C + \dots \text{ or } mu = \sum m_i u_i \quad (1.3)$$

$$mh = m_A h_A + m_B h_B + m_C h_C + \dots \text{ or } mh = \sum m_i h_i \quad (1.4)$$

$$\text{and } ms = m_A s_A + m_B s_B + m_C s_C + \dots \text{ or } ms = \sum m_i s_i \quad (1.5)$$

### 1.3 VOLUMETRIC ANALYSIS OF A GAS MIXTURE.

The analysis of a mixture of gases is often quoted by volume as this is the most convenient for practical determinations. Considered a volume  $V$  of a gaseous mixture at a temperature  $T$ , consisting of three constituents A, B and C. Let each of the constituents be compressed to a pressure  $P$  equal to the total pressure of the mixture, and let the temperature remain constant.

$$\text{Partial pressure of A, } p_A = \frac{m_A R_A T}{V} \quad \text{or} \quad m_A = \frac{p_A V}{R_A T}$$

$$\text{Also, the total pressure, } P = \frac{m_A R_A T}{V_A} \quad \text{or} \quad m_A = \frac{P V_A}{R_A T}$$

$$\text{Therefore, } p_A V = P V_A \quad \text{or} \quad V_A = \frac{p_A}{P} V$$

$$\text{In general, } v_i = \frac{P_i}{P} V \quad \text{or} \quad \frac{V_i}{V} = \frac{P_i}{P} \quad (1.6)$$

$$\text{and } \sum V_i = \frac{V}{P} \sum P_i \quad \text{since } \sum P_i = P,$$

$$\sum V_i = V \quad (1.7)$$

Therefore the volume of a mixture of gases is equal to the sum of the volumes of the individual constituents when each exists alone at the pressure and temperature of the mixture.

## Mixtures of Perfect Gases

Each of the constituents in the mixture is assumed to obey the perfect gas equation of state:

$$\text{for the mixture, } PV = mRT \quad \text{or } P = \frac{mRT}{V}$$

$$\text{for the constituent, } P_i V = m_i R_i T \quad \text{or } P_i = \frac{m_i R_i T}{V}$$

$$\text{From dalton's law, } P = \sum P_i = \sum \frac{m_i R_i T}{V}$$

$$\frac{mRT}{V} = \frac{T}{V} \sum m_i R_i$$

$$\text{that is, } mR = \sum m_i R_i \quad (1.8)$$

since  $R_i = \frac{R_o}{M_i}$  and  $R = \frac{R_o}{M}$ , therefore,  $\frac{mR_o}{M} = \sum \frac{m_i R_o}{M_i}$  and Eq. (1.8) becomes:

$$\frac{m}{M} = \sum \frac{m_i}{M_i} \quad (1.9)$$

The ratio of mass ( $m$ ) of a gas to its Molar mass ( $M$ ) is defined as number of moles contain by the volume of the gas, that is  $m/M = n$  and  $m_i/M_i = n_i$ . Substituting these in Eq. (1.9) we have:

$$n = \sum n_i \quad \text{or} \quad n = n_A + n_B + n_C + \dots \quad (1.10)$$

Therefore, the number of moles of a mixture is equal to the sum of the moles of the constituents.

### 1.4 THE MOLAR MASS AND SPECIFIC GAS CONSTANT

$$P_i V = m_i R_i T \quad \text{or} \quad P_i V = n_i R_o T \quad (1.11)$$

$$V \sum P_i = R_o T \sum n_i \quad (1.12)$$

since  $\sum P_i = P$ , and  $\sum n_i = n$

$$\text{therefore, } PV = nR_o T \quad (1.13)$$

This is a characteristic equation for the mixture, which shows that the mixture also acts as a perfect gas. It can be assumed that a mixture of perfect gases obeys all the perfect gas laws. The relationship between the volume fraction and pressure fraction is obtained from ratio of Eq. (1.12) to Eq. (1.13):

$$\frac{P_i V}{P V} = \frac{n_i R_o T}{n R_o T} \quad \text{or} \quad \frac{P_i}{P} = \frac{n_i}{n} \quad (1.14)$$

Combination of Eq. (1.14) with Eq. (1.6) gives:

$$\frac{P_i}{P} = \frac{n_i}{n} = \frac{V_i}{V} \quad (1.15)$$

In order to find the specific gas constant for the mixture in terms of the specific gas constants of the constituents, consider the following equations both for the mixture and for a constituent:

for mixture,  $PV = mRT$

for a constituent  $P_i V = m_i R_i T$  or  $\sum P_i V = \sum m_i R_i T$

then  $V \sum P_i = T \sum m_i R_i$

since  $\sum P_i = P$ ,  $PV = T \sum m_i R_i$  or  $mRT = T \sum m_i R_i$

therefore,  $mR = \sum m_i R_i$

$$R = \sum \frac{m_i}{m} R_i \quad (1.16)$$

where  $m_i/m$  is the mass fraction of a constituent.

## 1.5 SPECIFIC HEAT CAPACITIES OF A GAS MIXTURE

From the Gibbs-Dalton law,  $mu = \sum m_i u_i$

at constant volume,  $u = C_v T$

Therefore,  $mC_v T = \sum m_i C_{vi} T$

$$mC_v = \sum m_i C_{vi} \quad \text{or} \quad C_v = \sum [(m_i/m) C_{vi}] \quad (1.17)$$

Similarly,  $mh = \sum m_i h_i$

at constant pressure,  $h = C_p T$

Therefore,  $mC_p = \sum m_i C_{pi}$  or  $C_p = \sum [(m_i/m)C_{pi}]$  (1.18)

From Eqs. (1.17) and (1.18)

$$C_p - C_v = \sum [(m_i/m)C_{pi}] - \sum [(m_i/m)C_{vi}]$$

$$C_p - C_v = \sum [(m_i/m)(C_{pi} - C_{vi})]$$

Also,  $C_p - C_v = R_i$ , therefore,

$$C_p - C_v = \sum [(m_i/m)R_i]$$

Recall Eq. (1.16),  $R = \sum \frac{m_i}{m} R_i$ , therefore, for the mixture:

$$C_p - C_v = R$$

This shows that the following equations can also be applied to a mixture of gases:

$$\gamma = \frac{C_p}{C_v}; \quad C_v = \frac{R}{\gamma - 1}; \quad \text{and} \quad C_p = \frac{\gamma R}{\gamma - 1}.$$

## 2.0 FUELS AND COMBUSTION

In practical engines and power plants the source of heat is the chemical energy of substances called fuels. This energy is released during the chemical reaction of the fuel with oxygen. During combustion, the fuel elements combine with oxygen in an oxidation process which is rapid and is accompanied by the evolution of heat.

### 2.1 FUELS

A fuel is any substance which when ignited in the presence of oxygen or a substance containing oxygen such as air will liberate an appreciable quantity of heat energy. A fuel can be a solid, a liquid or a gas. It should be obtainable in abundant quantity, it should be relatively easy to handle and its combustion should allow satisfactory control.

The vast majority of fuels are based on carbon, hydrogen or some combination of carbon and hydrogen called hydrocarbons. These C-H combinations occur as solids liquids and gases. Coal, oils and natural gas are natural fuels that were laid down many millions of years ago, so they are often called fossil fuels.

#### 2.1.1 Solid Fuel

Coal is the most important solid fuel and the various types are divided into groups according to their chemical and physical properties. Most solid fuels are obtained by mining; they consist mainly of carbon, together with hydrogen, sulphur and some incombustible ash. According to their chemical and physical properties, those fuels are called anthracite, bituminous coal, brown coal, or peat.

***Analysis of Solid Fuels:*** A solid fuel is analyzed on a mass basis. In practice two types of analysis of a solid fuel such as coal are employed. They are ultimate analysis and proximate

analysis. *The ultimate analysis expresses the percentage by mass of the individual chemical elements in the solid fuel.* These elements usually include hydrogen, carbon, sulphur and nitrogen. *The proximate analysis gives the percentage composition by mass of coal into three main constituents namely: moisture, volatile matter and fixed carbon.*

### **2.1.2 Liquid Fuels**

Most liquid fuels are hydrocarbons, either saturated or unsaturated, which exist in the liquid phase at atmospheric conditions. There is no clear-cut boundary between gaseous and liquid fuels, because the one can be transformed into the other by changing the temperature and pressure. Also, petroleum oils are complex mixtures of sometimes hundreds of different fuels, but the necessary information to the engineer is the relative proportions of carbon, hydrogen, etc as given by the ultimate analysis. Examples of liquid fuels are petrol, kerosene and diesel oil.

***Analysis of Liquid Fuels:*** The ultimate analysis is used to determine the various constituents of a liquid fuel. The analysis gives the percentage content by mass of the liquid fuel in terms of chemical elements such as carbon, hydrogen, sulphur and ash.

### **2.1.3 Gaseous Fuels**

Gaseous fuels are chemically the simplest of the three groups. The main gaseous fuel in use occurs naturally but other gaseous fuels may be manufactured by the various treatments of coal. Examples of gaseous fuels are methane, butane, propane, ethane and producer gas.

***Analysis of Gaseous Fuels:*** The constituents of a gaseous fuel are usually analysed on a volume basis. Such analysis is called volumetric analysis. Since by Avogadro's Hypothesis, equal volumes of all gases contain the same number of molecules, under the same conditions

of temperature and pressure, it follows that analysis by volume (volumetric analysis) is the same as analysis by mole (Molar analysis).

$$\frac{P_i}{P} = \frac{n_i}{n} = \frac{V_i}{V} \quad (2.1)$$

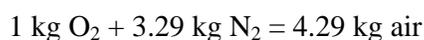
where  $V_i$  = volume of each constituent;  $V$  = total volume of all the constituents;  $n_i$  = number of moles of each constituent;  $n$  = total number of moles of all the constituents,  $P_i$  = partial pressure of each constituents; and  $P$  = total pressure of all the constituents.

## 2.2 AIR

For most combustion process, the oxygen is usually provided by atmospheric air. Air is mainly composed of oxygen, nitrogen and argon. The composition of air on a volume basis (or molar basis) is approximately 21% oxygen (O<sub>2</sub>), 78% nitrogen (N<sub>2</sub>) and 1% argon (Ar), the corresponding analysis on mass basis is 23.2% O<sub>2</sub>, 75.4% N<sub>2</sub> and 1.4% Ar. For all combustion calculations, it is customary to treat the argon and other traces of inert gases as nitrogen so that the composition of air is as shown in Table 2.1:

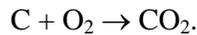
*Table 2.1: The composition of air*

| <b>Air content</b> | <b>Volume/Molar basis (%)</b> | <b>Mass basis (%)</b> |
|--------------------|-------------------------------|-----------------------|
| Oxygen             | 21                            | 23.3                  |
| Nitrogen           | 79                            | 76.7                  |

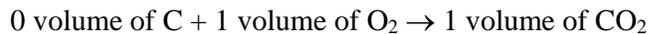
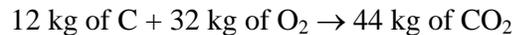


### 2.3 COMBUSTION EQUATIONS

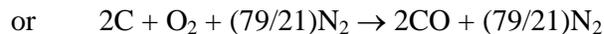
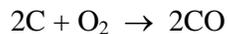
The first step in the description of a combustion process is the formulation of the chemical equation which shows how the atoms of the reactants are re-arranged to form the products. A chemical equation expresses the principle of the conservation of mass in terms of the conservation of atoms. Consider the chemical equation expressing the complete combustion of carbon and oxygen to carbon dioxide



Then, 1 kmol of C + 1 kmol of O<sub>2</sub> → 1 kmol of CO<sub>2</sub>

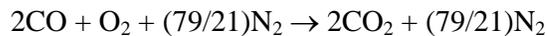


The volume of carbon is written as zero since the volume of a solid is negligible in comparison with that of a gas. Consider incomplete combustion of carbon, which occurs when there is an insufficient supply of oxygen to burn the carbon completely to carbon dioxide.



i.e., 24 kg of C + 32 kg O<sub>2</sub> + 105.3 kg of N<sub>2</sub> = 56 kg of CO + 105.3 kg of N<sub>2</sub>

If a further supply of oxygen is available, then the combustion can continue to completion.



By mass, 56 kg of CO + 32 kg of O<sub>2</sub> + 105.3 kg of N<sub>2</sub> = 88 kg of CO<sub>2</sub> + 105.3 kg of N<sub>2</sub>

or 1 kg of CO + (32/56)kg of O<sub>2</sub> + (105.3/56)kg of N<sub>2</sub> = (88/56)kg of CO<sub>2</sub> + (105.3/56)kg of N<sub>2</sub>

### 2.4 STOICHIOMETRIC AIR- FUEL RATIO

A stoichiometric mixture of air and fuel is one that contains just sufficient oxygen for the complete combustion of the fuel.

**Stoichiometric air fuel ratio (SAF):** is the ratio of the amount of stoichiometric air to the amount of fuel in a combustion process.

$$SAF = \frac{\text{Amount of stoichiometric air}}{\text{Amount of Fuel}} \quad (2.2)$$

**Stoichiometric Air:** This is the quantity of air which contains the minimum amount of oxygen theoretically required for complete combustion of the fuel.

**Excess Air:** Any air in excess of the stoichiometric air is regarded as excess air.

**Actual Air:** This is the sum of the stoichiometric air and excess air supplied to the fuel for complete combustion.

**Actual Air-Fuel Ratio (AAF):** This is the ratio of this amount of actual air to the amount of fuel in a combustion process.

**Weak Mixture:** A mixture of air and fuel which has excess supply of air is referred to as a weak mixture.

**Rich mixture:** This is a mixture of air and fuel in which less than the stoichiometric air is supplied, that is, there is a deficiency in the amount of air hence some un-burnt fuel elements appear as products. In this case, the combustion is said to be incomplete.

**Percentage of Excess Air:** The percentage of excess air is given by this following:

$$\text{Percentage of Excess Air} = \frac{AAF - SAF}{SAF} \times 100\% \quad (2.3)$$

For example, a 20% excess air is equivalent to 120% stoichiometric air or 1.20 of stoichiometric air.

For gaseous fuels the ratios are expressed by volume and for solid and liquid fuels the ratios are expressed by mass. Eq. (2.3) gives a positive result when the mixture is weak and a negative result when the mixture is rich.

**Mixture Strength:** The mixture strengths is define as the ratio of stoichiometric air-fuel ratio to actual air-fuel ratio

$$\text{Mixture Strength} = \frac{SAF}{AAF} \times 100\% \quad (2.4)$$

The result of Eq. (2.4) is less than 100% when the mixture is weak and greater than 100% when the mixture is rich.

## 2.5 EXHAUST AND FUEL GAS ANALYSIS

The analysis of the products of combustion may be quoted on a volume or a mass basis. In either case, if the water vapour is assumed to be present as it is in the hot exhaust gases from an engine, the analysis is called a wet analysis. When the vapour is assumed to be condensed and removed, it is called a dry analysis.

## 2.6 CALORIFIC VALUES OF FUELS

The calorific value (also called heating value) of a fuel *is the amount of heat energy liberated by the complete burning of a unit quantity of the fuel in air*. Calorific values are normally expressed as kJ/kg of fuel, kJ/m<sup>3</sup> of fuel or kJ/mol of fuel as may be found convenient. Two types of calorific value namely: the higher (or gross) calorific value and the lower (or net) calorific value are defined depending on the phase the H<sub>2</sub>O is present in the products of combustion.

***The Higher Calorific Value:*** Is the calorific value of the fuel when the H<sub>2</sub>O in the products of combustion is present in the liquid phase. The higher calorific value is designated by the symbol *HCV* (kJ/kg) for solid and liquid fuels, and  $\overline{HCV}$  (kJ/m<sup>3</sup>) for gaseous fuels.

***The Lower Calorific Value:*** Is the calorific value of the fuel when H<sub>2</sub>O in the products is present in the vapour phase. The lower calorific value is represented by the symbol *LCV* (kJ/kg) for solid and liquid fuels, and  $\overline{LCV}$  (kJ/m<sup>3</sup>) for gaseous fuels.

If  $m_f$  = mass of a solid or liquid fuel (kg); and  $V_f$  = Volume of a gaseous fuel ( $m^3$ ), then the total heat liberated is given as:

(i) For solid and liquid fuels:  $Q_g = m_f(HCV)$  and  $Q_n = m_f(LCV)$

(ii) For gaseous fuels:  $Q_g = V_f(\overline{HCV})$  and  $Q_n = V_f(\overline{LCV})$

The higher calorific value is determined for a solid or liquid fuel in a bomb calorimeter where combustion occurs at constant volume and for a gaseous fuel, it is determined in a gas calorimeter at constant pressure by burning the gas in a stream of air under steady flow conditions. Both the higher calorific value and lower calorific value are related by the following equations.

(i) At constant volume (for solid and liquid fuels)

$$Q_{g,v} = m_f(HCV)_v = m_f(LCV)_v + m_c U_{fg}$$

(ii) At constant pressure (for gaseous fuels)

$$Q_{g,p} = V_f(\overline{HCV})_p = V_f(\overline{LCV})_p + m_c h_{fg}$$

where  $m_c$  is the mass of condensates/condensed water vapour per unit quantity of fuel (kg);  $U_{fg}$  and  $h_{fg}$  are the latent heats of moisture formed by combustion at constant volume and constant pressure, respectively.

Values of  $U_{fg}$  and  $h_{fg}$  are retrieved from steam tables at a standard reference temperature of 25°C.

## 2.7 POWER PLANT THERMAL EFFICIENCY

### 2.7.1 The Overall Efficiency

This is a criterion of performance used to measure the proportion of the latent energy in the fuel that is converted to mechanical work. It is defined as the ratio of the net work output to latent energy of fuel.

$$\eta_{overall} = \frac{\text{Net work output}}{\text{Latent energy of fuel}} = \frac{W_{net}}{Q_{g,p}} \text{ or } \frac{W_{net}}{Q_{n,p}}$$

The overall efficiency can also be expressed as a product of combustion efficiency,  $\eta_{comb}$  and the cycle efficiency,  $\eta_{cycle}$ .

### 2.7.2 The Combustion Efficiency

The combustion efficiency,  $\eta_{comb}$  determines the proportion of latent energy of the fuel which is transferred as heat energy to the working fluid and is defined as the ratio of heat input to latent energy of fuel.

$$\eta_{comb} = \frac{\text{Heat transferred to working fluid}}{\text{Latent energy of fuel}} = \frac{Q}{Q_{g,p}} \text{ or } \frac{Q}{Q_{n,p}}$$

### 2.7.3 The Cycle Thermal Efficiency

It is often called the thermal efficiency is a criterion of performance which determines how much of the heat input to a system undergoing a thermodynamic cycle is converted to net work output. It is defined as the ratio of the net work output to the heat input.

$$\eta_{cycle} = \frac{\text{Net work output}}{\text{Heat transferred to working fluid}} = \frac{W_{net}}{Q}$$

Therefore,  $\eta_{overall} = \eta_{comb} \times \eta_{cycle}$

### 3.0 AVAILABILITY OR EXERGY

#### 3.1 INTRODUCTION

Availability or Exergy is a measure of the maximum useful work that can be done by a system interacting with an environment which is at a constant pressure  $P_o$  and temperature  $T_o$ . The simplest case to consider is that of a heat reservoir (i.e. a heat source of infinite capacity and therefore invariable temperature) of temperature  $T$ . The maximum efficiency with which heat withdrawn from a reservoir may be converted into work is the Carnot efficiency. Therefore, the availability of a reservoir at  $T$  providing a rate of heat transfer  $Q$ , in surroundings at  $T_o$ , is  $Q(T - T_o)/T$ .

#### 3.2 AVAILABILITY OF NON-FLOW SYSTEMS

Consider a system consisting of a fluid in a cylinder behind a piston, the fluid expanding reversibility from initial conditions of  $P_1$  and  $T_1$  to final atmospheric conditions of  $P_o$  and  $T_o$ . Imagine also that the system works in conjunction with a reversible heat engine which receives heat reversibility from the fluid in the cylinder such that the working substance of the heat engine follows the cycle 0-1-A-0 as shown in Fig. 3.1, where  $s_1 = s_A$  and  $T_o = T_A$ .

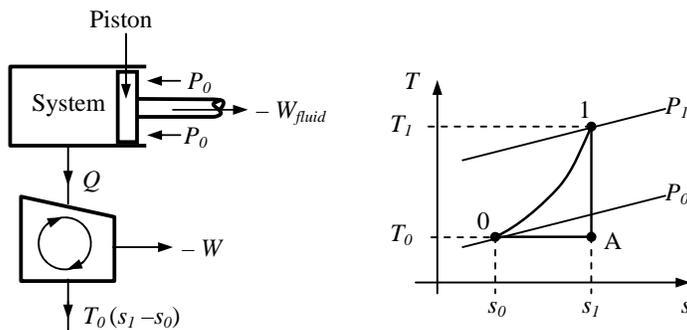


Fig. 3.1: A reversible heat engine receiving heat from a cylinder

The work done by this engine is:

$$-W = Q - T_0(s_1 - s_0)$$

The heat supplied to the engine is equal to the heat rejected by the fluid in the cylinder.

Therefore for the fluid in the cylinder undergoing the process 1 to 0, we have:

$$Q = (u_1 - u_0) + W_{fluid}$$

$$W_{fluid} = -(u_1 - u_0) + Q$$

$$\therefore W_{fluid} = (u_0 - u_1) + Q$$

Therefore the total work output is given by (Fig. 3.1):

$$-W_{fluid} - W = -(u_0 - u_1) - Q + Q - T_0(s_1 - s_0)$$

$$\text{or } -W_{fluid} - W = (u_1 - u_0) - T_0(s_1 - s_0) \quad (3.1)$$

The work done by the fluid on the piston is less than the total work done by the fluid,  $-W_{fluid}$ ,

since there is work done on the atmosphere which is at the constant pressure  $P_0$ . That is,

Work done by fluid on atmosphere,

$$W_{am} = -P_0(V_0 - V_1) \quad (3.2)$$

Therefore, work done by fluid =  $-W_{fluid} + W_{am}$ .

Max. work available = work done by fluid + Work done by engine

$$= -W_{fluid} + W_{am} - W$$

$$\text{or } \text{Max. work available} = -W_{fluid} - W + W_{am} \quad (3.3)$$

Substituting Eqs. (3.1) and (3.2) in Eq. (3.3) we have:

$$\begin{aligned} \text{Max. work available} &= (u_1 + P_0 V_1 - T_0 s_1) - (u_0 + P_0 V_0 - T_0 s_0) \\ &= a_1 - a_0 \end{aligned} \quad (3.4)$$

Where  $a = u + P_0 V - T_0 s$  and is called the specific non-flow exergy.

### 3.3 AVAILABILITY AND STEADY-FLOW SYSTEMS

Let fluid flow steadily with a velocity  $v_1$  from a reservoir to which the pressure and temperature remain constant at  $P_1$  and  $T_1$  through an apparatus to atmospheric pressure  $P_0$ . Let the reservoir be at a height  $z_1$  from the datum, which can be taken at exit from the apparatus, i.e.,  $z_0 = 0$ . For a maximum work output to be obtained from the apparatus the exit velocity,  $v_0$ , must be zero. It can be shown as for non-flow systems above that a reversible heat engine working between the limits would reject  $T_0(s_1 - s_0)$  units of heat, where  $T_0$  is the atmospheric temperature.

Therefore, we have:

$$\text{Specific exergy} = (h_1 + \frac{1}{2}v_1^2 + z_1g) - h_0 - T_0(s_1 - s_0)$$

In many thermodynamic systems the kinetic and potential energy terms are negligible, therefore,

$$\text{Specific exergy} = (h_1 - T_0s_1) - (h_0 - T_0s_0) \quad (3.5)$$

$$= b_1 - b_0 \quad (3.6)$$

where:  $h - T_0s$

Specific exergy can be written as;  $h_1 - h_0 + T_0(s_0 - s_1)$

### 3.4 EFFECTIVENESS

Instead of comparing a process to some imaginary ideal process, as is done in the case of isentropic efficiency, it is a better measure of the usefulness or effectiveness of the process to compare the useful output of the process with the loss of exergy of the system. The useful output of a system is equal to the increase of exergy of the surroundings. Therefore, expansion or cooling process, the effectiveness ( $\varepsilon$ ) of the system is the ratio of the increase of exergy of the surroundings to the loss of exergy of the system:

$$\varepsilon = \frac{\text{increase of exergy of the surroundings}}{\text{loss of exergy of the system}} \quad (3.7)$$

For a compression or heating process the effectiveness ( $\varepsilon$ ) becomes

$$\varepsilon = \frac{\text{increase of exergy of the system}}{\text{loss of exergy of the surroundings}} \quad (3.8)$$

## 4.0 THE HEAT ENGINE CYCLES

### 4.1 INTRODUCTION

#### *A cycle*

If a substance passes through a series of processes such that it is eventually returned to its original state, the substance is said to have been taken through a cycle. During a cycle there will be some heat transfer and some work transfer to and from the substance. After performing a cycle, the substance is returned to its original state, so by the first law of thermodynamics.

$$\text{Net heat transfer} = \text{Net work done}$$

#### *Net work done*

The net work transfer can be determined by an analysis of the net heat transfer for a cycle. Net work done ( $\oint_W$ ) is the difference between net heat received and net heat rejected, or the difference between positive work done and negative work done therefore,

$$\oint_W = \text{net heat received} - \text{net heat rejected} \quad (4.1)$$

or  $\oint_W = \text{positive work done} - \text{negative work done}$

The net work done can also be determined from pressure-volume (P-V) diagram. The area under a P-V diagram is equal to the work done, therefore, the net work done of a cycle is equal to the net area of a P-V diagram of the cycle.

#### **Theoretical or ideal thermal efficiency**

The theoretical or ideal thermal efficiency ( $\eta_{th}$ ) for a cycle is the ratio of the net work done to the net heat transfer and is given as:

Thermal efficiency,  $\eta_{th} = \frac{\text{Net work done}}{\text{Net heat received}}$  (4.2)

or  $\eta_{th} = \frac{\oint W}{\oint Q} = \frac{\text{Heat received} - \text{Heat rejected}}{\text{Heat received}}$

### **Actual thermal efficiency and Relative efficiency**

The actual thermal efficiency ( $\eta_{actual}$ ) of a practical cycle is the ratio of the actual work done to the thermal energy from fuel and is given as:

$$\eta_{actual} = \frac{\text{Actual work done}}{\text{Thermal energy from fuel}} \quad (4.3)$$

This is always less than theoretical thermal efficiency. The ratio of the actual thermal efficiency to the ideal thermal efficiency is called the relative efficiency ( $\eta_{rel}$ ) or efficiency ratio, thus

$$\eta_{rel} = \frac{\text{Actual thermal efficiency}}{\text{Ideal thermal efficiency}} \quad (4.4)$$

### **Work Ratio**

Work ratio is defined as the ratio of net work done to the positive work done.

$$\text{Work ratio} = \frac{\text{Net work done}}{\text{Positive work done}} \quad (4.5)$$

### **Specific steam consumption**

Specific steam consumption (SSC) is the mass of steam used per unit power output in kg/kWh. Therefore,

$$SSC = \frac{\text{Mass flow of steam kg/h}}{\text{Power output in kW}} \quad (4.6)$$

In the case of internal combustion engines, such as the gas turbine and the petrol or diesel engine, the specific fuel consumption (SFC) is used, therefore,

$$SFC = \frac{\text{Mass flow of fuel kg/h}}{\text{Power output in kW}} \quad (4.7)$$

The cycle which has a lower specific steam or fuel consumption indicates that it has better energy conversion performance than a cycle with a higher specific steam or fuel consumption.

## 4.2 THE CARNOT CYCLE

It can be shown from the second law of thermodynamics that no heat engine can be more efficient than a reversible heat engine working between the same temperature limits. Carnot showed that the most efficient possible cycle is the one in which all the heat supplied is supplied at one fixed temperature, and all the heat rejected is rejected at a lower fixed temperature. Therefore, the cycle is sometimes referred to as the *constant temperature cycle*, since heat is received and rejected during isothermal operations only. The cycle consists of two isothermal processes joined by two adiabatic processes. Since all processes are reversible, then the adiabatic process in the cycle are also isentropic, that is, *frictionless* or *reversible adiabatic* (Fig. 4.1).

Process 1 to 2 is isentropic expansion from  $T_1$  to  $T_2$

Process 2 to 3 is isothermal heat rejection  $T_2 = T_3$

Process 3 to 4 is isentropic compression from  $T_3$  to  $T_4$  or  $T_2$  to  $T_1$

Process 4 to 1 is isothermal heat addition  $T_1 = T_4$

No practical engine was ever built to run on this cycle.

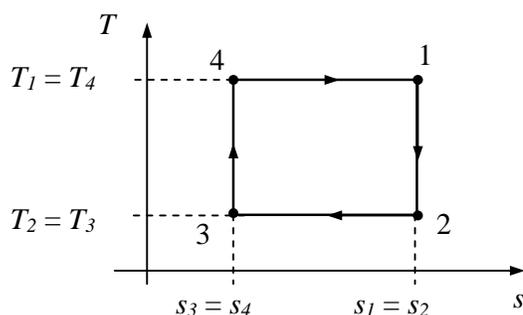


Fig. 4.1: Carnot cycle.

Carnot cycle is important because it is composed of reversible processes, which are the most thermodynamically efficient processes, so its thermal efficiency establishes the maximum thermal efficiency possible within the temperature limits of the cycle. From the T-S diagram (Fig. 4.1):

Heat received from 4 to 1 =  $T_1(s_1 - s_4)$  or Area Under 4 -1

Heat rejected from 2 to 3 =  $T_2(s_2 - s_3)$  or Area under 2-3

Thermal efficiency,  $\eta_{th} = \frac{\text{Heat received} - \text{Heat rejected}}{\text{Heat received}}$

Therefore, thermal efficiency for Carnot cycle is:

$$\eta_{carnot} = \frac{T_1(s_1 - s_4) - T_2(s_2 - s_3)}{T_1(s_1 - s_4)}$$

$$\eta_{carnot} = \frac{(T_1 - T_2)(s_1 - s_4)}{T_1(s_1 - s_4)} \quad \text{since } s_1 = s_2, \text{ and } s_3 = s_4$$

$$\eta_{carnot} = \frac{T_1 - T_2}{T_1} \text{ or } 1 - \frac{T_2}{T_1} \quad (4.8)$$

$$\eta_{carnot} = \frac{\text{max .abs.temp.} - \text{min .abs.temp.}}{\text{max .abs.temp.}}$$

Net work done = *heat received* – *heat rejected*

$$\oint W = (T_1 - T_2)(s_1 - s_4)$$

### 4.3 THE CARNOT CYCLE FOR A PERFECT GAS

A Carnot cycle for a perfect gas is shown on both  $P$ - $V$  and  $T$ - $s$  diagrams in Fig. 4.2. It consists of four reversible processes.

(a) *1-2 Adiabatic Expansion*

Pressure falls from  $P_1$  to  $P_2$

Volume increases from  $V_1$  to  $V_2$

Temperature falls from  $T_1$  to  $T_2$

$$\text{Work done} = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

No heat transfer during adiabatic process, therefore,  $Q = 0$

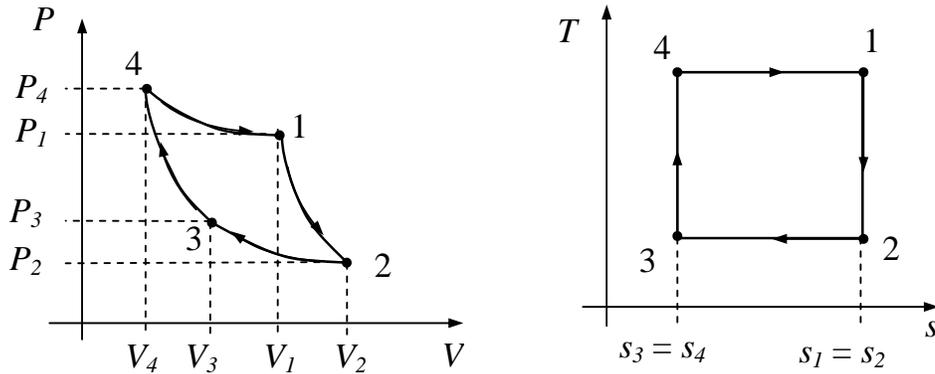


Fig. 4.2: Carnot cycle for a perfect gas.

(b) 2-3 Isothermal compression (Heat rejection)

Pressure increases from  $P_2$  to  $P_3$

Volume reduced from  $V_2$  to  $V_3$

Temperature remains constant at  $T_2 = T_3$

$$\text{Work done} = P_2V_2 \ln(V_3/V_2) = -P_2V_2 \ln(V_2/V_3)$$

$$= -mRT_2 \ln(V_2/V_3)$$

For an isothermal process,  $Q = W$

$$\therefore \text{Heat rejected} = mRT_2 \ln(V_2/V_3)$$

Note that  $P_2V_2 = P_3V_3 = C$  during this process

(c) 3-4 Adiabatic Compression

Pressure increases from  $P_3$  to  $P_4$

Volume reduced from  $V_3$  to  $V_4$

Temperature increases from  $T_3$  to  $T_4$

$$\text{Work done} = \frac{P_3V_3 - P_4V_4}{\gamma - 1} = -\frac{P_4V_4 - P_3V_3}{\gamma - 1} = -\frac{mR(T_4 - T_3)}{\gamma - 1}$$

No heat transfer during adiabatic process, therefore,  $Q = 0$

(d) *4-1 Isothermal Expansion (Heat supply)*

Pressure falls from  $P_4$  to  $P_1$

Volume increases from  $V_4$  to  $V_1$

Temperature remains constant at  $T_4 = T_1$

$$\text{Work done} = P_4V_4 \ln(V_1/V_4) = mRT_4 \ln(V_1/V_4) = mRT_1 \ln(V_1/V_4)$$

For an isothermal process  $Q = W$

$$\text{Heat supply} = mRT_4 \ln(V_1/V_4)$$

Note that  $P_4V_4 = P_1V_1 = C$  during this process

The net work done during this cycle may be determined by summing the work done in the various processes, taking the expansions as positive work and the compressions as negative work. Therefore,

Net work done/cycle =  $\oint W$

$$\oint W = \frac{mR(T_1 - T_2)}{\gamma - 1} - mRT_2 \ln \frac{V_2}{V_3} - \frac{mR(T_4 - T_3)}{\gamma - 1} + mRT_1 \ln \frac{V_1}{V_4}$$

Now  $\frac{mR(T_1 - T_2)}{\gamma - 1} = \frac{mR(T_4 - T_3)}{\gamma - 1}$  since  $T_1 = T_4$  and  $T_2 = T_3$

$$\text{Hence, } \oint W = mRT_1 \ln \frac{V_1}{V_4} - mRT_2 \ln \frac{V_2}{V_3} \quad (4.9)$$

From adiabatic process 1-2

$$\frac{T_1}{T_2} = \left( \frac{V_2}{V_1} \right)^{\gamma - 1} = r_v^{\gamma - 1}$$

(4.10a)

From adiabatic process 3-4

$$\frac{T_4}{T_3} = \left( \frac{V_3}{V_4} \right)^{\gamma-1} = r_v^{\gamma-1}$$

(4.10b)

where  $r_v$  = adiabatic compression and expansion volume ratio

$$\frac{T_1}{T_2} = \frac{T_4}{T_3} \text{ since } T_1 = T_4 \text{ and } T_2 = T_3$$

Therefore, from Eqs. (4.10a) and (4.10b)

$$\frac{V_2}{V_1} = \frac{V_3}{V_4} \text{ or } \frac{V_1}{V_4} = \frac{V_2}{V_3}$$

(4.11)

Substituting Eq. (4.11) in Eq. (4.9):

$$\oint W = mRT_1 \ln \frac{V_1}{V_4} - mRT_2 \ln \frac{V_1}{V_4}$$

$$\text{Net work done, } \oint W = mR \ln \frac{V_1}{V_4} (T_1 - T_2) \quad (4.12)$$

The positive work done ( $W_{+ve}$ ) during the Carnot cycle occurs during processes 4-1 and 1-2:

$$\text{Positive work done, } W_{+ve} = mRT_1 \ln \frac{V_1}{V_4} + \frac{mR(T_1 - T_2)}{\gamma - 1} \quad (4.13)$$

$$\text{Work ratio} = \frac{\text{Net work done}}{\text{Positive work done}} = \frac{\oint W}{W_{+ve}}$$

$$\eta_{carnot} = \frac{\text{Heat received} - \text{Heat rejected}}{\text{Heat received}}$$

$$\eta_{carnot} = \frac{mRT_1 \ln \left( \frac{V_1}{V_4} \right) - mRT_2 \ln \left( \frac{V_2}{V_3} \right)}{mRT_1 \ln \left( \frac{V_1}{V_4} \right)}$$

$$\eta_{carnot} = \frac{mR \ln \left( \frac{V_1}{V_4} \right) (T_1 - T_2)}{mR \ln \left( \frac{V_1}{V_4} \right) T_1} \text{ since } \frac{V_1}{V_4} = \frac{V_2}{V_3}$$

$$\eta_{carnot} = \frac{T_1 - T_2}{T_1} \text{ or } 1 - \frac{T_2}{T_1} \text{ and from Eq. (4.10a), } \frac{T_2}{T_1} = \frac{1}{r_v^{\gamma-1}}$$

Therefore,  $\boxed{\eta_{carnot} = 1 - \frac{1}{r_v^{\gamma-1}}}$  (4.14)

#### 4.4 MEAN EFFECTIVE PRESSURE

For internal combustion engines work ratio is not such a useful concept; since the work done on and by the working fluid takes place inside one cylinder. In order to compare reciprocating engines another term is defined called the mean effective pressure. *The mean effective pressure ( $P_m$ ) is defined as the height of a rectangle having the same length and area as the cycle plotted on a P-V diagram.*

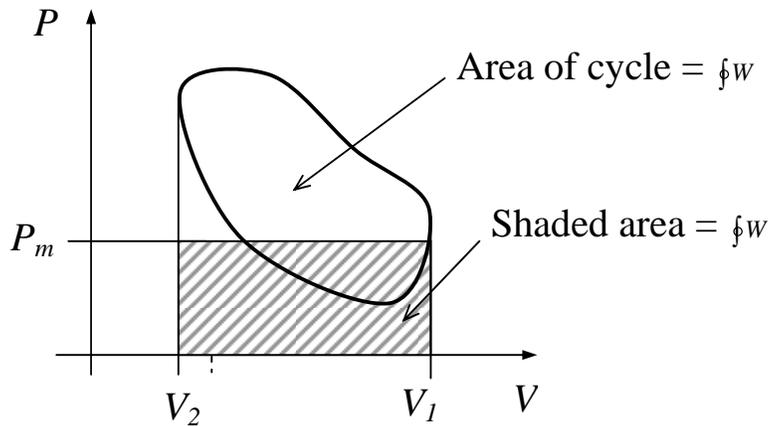


Fig. 4.4: Mean effective pressure

From Fig. 4.4, shaded area = area of cycle

therefore,  $P_m \times (V_1 - V_2) = \oint W$

Mean Effective pressure,  $P_m = \frac{\oint W}{V_1 - V_2} = \frac{\text{Net work done}}{\text{Swept volume}}$

where  $(V_1 - V_2) =$  swept volume of the cylinder

Therefore, Mean effective pressure ( $P_m$ ) can also be defined as the work done per unit swept volume.

#### 4.5 THE CONSTANT PRESSURE CYCLE (JOULE CYCLE)

In this cycle the heat supply and heat rejection processes occur reversibly at constant pressure. The expansion and compression processes are isentropic. The cycle is shown on a  $P$ - $V$  and  $T$ - $s$  diagrams in Fig. 4.5. This cycle was at one time used as the ideal basis for a hot-air reciprocating engine, and the cycle has known as the Joule or Brayton cycle.

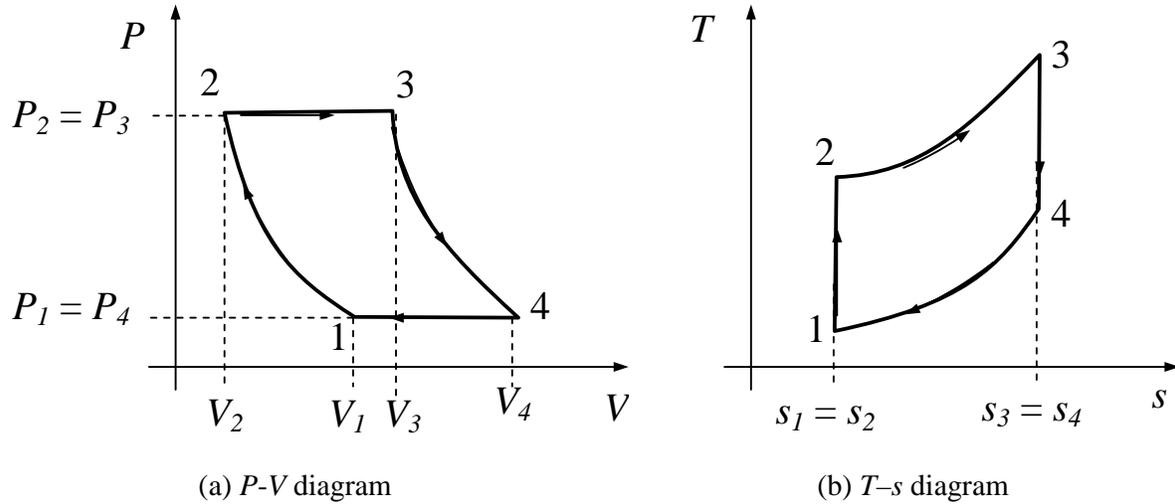


Fig. 4.5: Constant pressure (Joule) cycle.

Referring to Fig. 4.5, the following is the analysis of the four processes:

Assume that  $P_1$ ,  $V_1$ ,  $T_1$  and the volume ratio  $V_1/V_2$  or  $r_v$  are known

*Process 1-2: Adiabatic compression*

$$PV^\gamma = C, \text{ and } s_1 = s_2$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \text{ or } \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\therefore T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1 r_v^{\gamma-1} \quad (4.15)$$

where  $r_v = V_1/V_2 =$  adiabatic compression volume ratio

$$\text{Also, } P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$P_2 = P_1(V_1/V_2)^\gamma = P_1 r_v^\gamma \quad (4.16)$$

$$\text{Work done (compression), } W_{1-2} = -\frac{P_2 V_2 - P_1 V_1}{\gamma - 1} \quad (4.17)$$

$$\text{Heat rejected, } Q_{1-2} = 0$$

*Process 2-3: Constant pressure expansion (heat addition)*

$$P_2 = P_3 \quad \text{and} \quad \frac{V_2}{T_2} = \frac{V_3}{T_3}$$

Substitute for  $T_2$  (Eq. 4.15). Hence,

$$T_3 = T_2 \left( \frac{V_3}{V_2} \right) = \left( \frac{V_3}{V_2} \right) T_1 r_v^{\gamma-1} \quad (4.18)$$

$$\text{Work done} = P_2(V_3 - V_2) \quad (4.19)$$

$$\text{Heat received, } Q_{2-3} = mC_p(T_3 - T_2)$$

*Process 3-4: Adiabatic expansion*

$$PV^\gamma = C, \text{ and } s_3 = s_4$$

For adiabatic processes 1-2 and 3-4, both have the same pressure ratios

$$\frac{P_2}{P_1} = \frac{P_3}{P_4} \quad \therefore \quad \frac{P_2}{P_1} = \left( \frac{V_1}{V_2} \right)^\gamma \quad \text{and} \quad \frac{P_3}{P_4} = \left( \frac{V_4}{V_3} \right)^\gamma$$

$$\text{Hence, } \frac{V_1}{V_2} = \frac{V_4}{V_3} = r_v \quad (4.20)$$

$$\text{and } \frac{V_4}{V_1} = \frac{V_3}{V_2} \quad (4.21)$$

$$\frac{T_4}{T_3} = \left( \frac{V_3}{V_4} \right)^{\gamma-1}, \quad \therefore \quad T_4 = T_3 \left( \frac{V_3}{V_4} \right)^{\gamma-1}$$

Substitute for  $r_v$  (Eq. 4.20)

$$\therefore \quad T_4 = T_3 r_v^{-(\gamma-1)}$$

Substitute for T3 (Eqn. 4.18)

$$T_4 = r v^{(\gamma-1)}$$

Heat received,  $Q_{2-3} = 0$

Process 4-1: Constant pressure heat rejection,  $P_4 = P_1$

Work done =  $-P_1(V_4 - V_1)$

Heat rejected =  $mC_p(T_4 - T_1)$

The net work-done during the cycle will be the sum of the work done during these processes, taking expansions as positive work and the compressions as negative work.

Alternatively, the work done may be obtained from:

$$= \text{Heat received} - \text{Heat rejected}$$

In this cycle, heat is received during constant pressure process 2-3 and rejected during constant pressure process 4-1. No heat is received or rejected during the adiabatic processes.

$$\text{Thermal efficiency, } \eta = \frac{\text{Heat received} - \text{Heat rejected}}{\text{Heat received}}$$

Work done (W) = Heat received x Thermal efficiency,

Also heat received or rejected = change of enthalpy

Heat received =  $h_3 - h_2$

and Heat rejected =  $h_4 - h_1$

#### **4.6 THE AIR STANDARD CYCLES**

Air standard cycles are theoretical power cycles which form the theoretical basis for the analysis of reciprocating internal combustion (I,C) engines. Although, in the actual power cycle, the working fluid consists of air and fuel, for simplicity of analysis, the fluid is assumed

to be air which is used respectively in the theoretical cycle. There are several air standard cycles which may engines, some of the cycles are: Otto, Diesel and dual-combustion cycles. The related efficiencies of these cycles are called air standard efficiencies.

#### 4.7 THE OTTO CYCLE (CONSTANT VOLUME CYCLE)

The Otto cycle is the ideal air standard cycle for the petrol engine (Spark-ignition engine), and high speed oil engine (compression –ignition engine), The four non-flow processes comprising a complete cycle are shown in Fig. 4.5. The swept volume is equal to  $m(v_1 - v_2)$ , where  $m$  is the mass of fluid in the cylinder. The processes are as follows:

Fig. 4.5: The Otto Cycle.

Process 1-2: Air is compressed adiabatically through a volume ratio  $V_1/V_2$  known as the compression ratio,  $r_v$ .

Process 2-3: Constant volume heat addition  $V_2 = V_3$ .

Process 3-4: Air is expanded adiabatically to the original volume

Process 4-1: Constant volume heat rejection  $V_4 = V_1$ .

The work done during the cycle maybe obtained as follows

- (i) The workdone during the constant volume processes, 2-3 and 4-1 are zero
- (ii) The work done during adiabatic expansion 3 – 4 is positive
- (iii) The work done during adiabatic compression is negative
- (iv) The net workdone is the sum of the work done by these two adiabatic processes.

Alternatively, the cycle work done can be determined using

$$\text{Heat received} - \text{Heat rejected.}$$

In this cycle heat is received and rejected only during constant volume processes.

Heat received =  $mC_v(T_3 - T_2)$  and

Heat rejected =  $mC_v(T_4 - T_1)$

#### 4.8 THE DIESEL CYCLE

The air standard Diesel cycle forms the theoretical basis for the analysis of compression ignition (IC) engines (Diesel engines). The Diesel cycle is quite similar to the Otto cycle except that the heat addition occurs at constant pressure instead of constant volume. The cycle (Fig. 4.6) consists of two adiabatic processes, 1-2 and 3-4, one constant pressure process, 2-3, and one constant volume process, 4-1.

The work done during the cycle may be determined as follows:

- (i) Processes 2-3 and 3-4 are expansions and hence give positive workdone
- (ii) Process 1-2 is a compression and hence will give negative workdone
- (iii) The net work done during the cycle will be the sum of the workdone during these processes, addition of eqns. (4.34), (4.34) and (4.36).

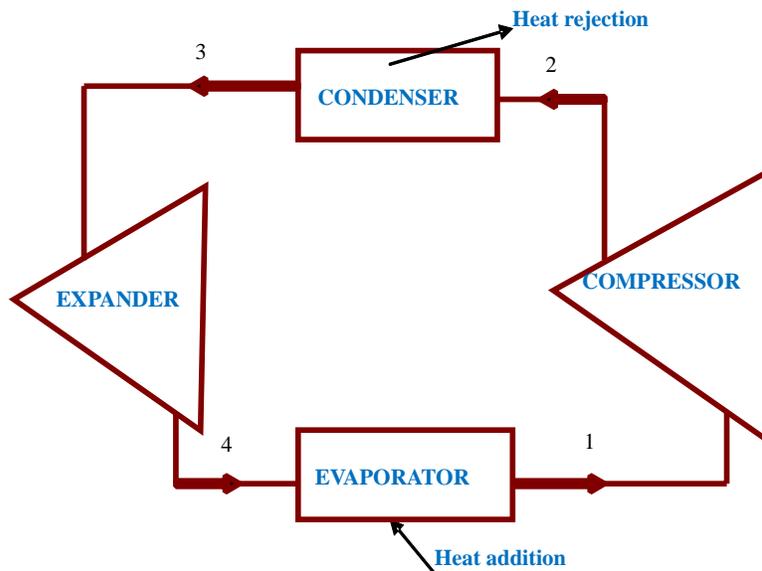
Alternatively, the work done may be obtained from the difference between the heat received during constant pressure process, 2-3 and the heat rejected during constant volume process 4-

$$= \text{Heat received} - \text{Heat rejected}$$

## 5.0 REFRIGERATION CYCLES

The vapour compression cycle is the most widely used refrigeration cycle in practice. In this cycle, a compressor compresses the refrigerant to a higher pressure and temperature from an evaporated vapour at low pressure and temperature. The compressed refrigerant is condensed into liquid form by releasing the latent heat of condensation to the condenser water. Liquid refrigerant is then throttled to a low-pressure, low temperature vapour, producing the refrigeration effect during evaporation. Vapour compression is often called mechanical refrigeration, that is, refrigeration by mechanical compression.

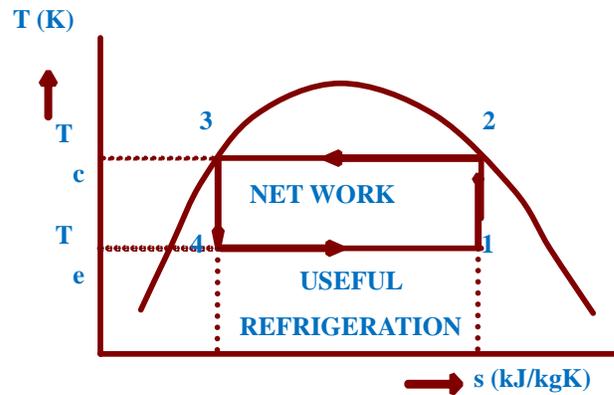
### 5.1 CARNOT REFRIGERATION CYCLE



**Fig. 5.1 Carnot Refrigeration cycle**

The Carnot cycle is one whose efficiency cannot be exceeded when operating between two given temperature. The Carnot heat engine receives energy at a high of temperature, converts a portion of the energy into work, and discharges the remainder to a heat sink a low level of temperature.

The Carnot refrigeration cycle performs the reverse effect of the heat engine, because it transfers energy from a low level of temperature to a high level of temperature. The diagram of the equipment and the temperature-entropy diagram of the refrigeration cycle are shown in Figs 5.1 and 5.2.



**Fig. 5.2: T-s diagram of the Carnot refrigeration cycle**

The processes which constitute the cycle are:

Process 1 - 2: Isentropic compression,  $S_1 = S_2$

Process 2 - 3: Isothermal rejection of heat  $T_c = \text{constant}$  i.e.  $T_2 = T_3$

Process 3 - 4: Isentropic expansion  $S_3 = S_4$

Process 4 - 1: Isothermal addition of heat (heat absorption from the cold reservoir) at  $T_e = \text{constant}$  i.e.  $T_1 = T_4$

All processes in the Carnot cycle are thermodynamically reversible. Processes 1-2 and 3-4 are consequently reversible adiabatic (isentropic).

The withdrawal of heat from the low temperature source in process 4-1 is the refrigeration step and is the entire purpose of the cycle. All the other processes in the cycle function so that the low temperature energy can be discharged to some convenient high-temperature sink.

The Carnot cycle, consist of reversible which make its efficiency high than could be achieved in an actual cycle. Although Carnot cycle is an unattainable ideal cycle, it necessary to study the cycle because of the following reason:

- (i) It serves a standard of comparison, and
- (ii) It provides a convenient guide to the temperatures that should be maintained to achieve maximum effectiveness.

### 5.3 COEFFICIENT OF PERFORMANCE (COP)

The index of performance is not called efficiency, because that term is usually reserved for the ratio of output to input. The ratio of output to input would be misleading applied to a refrigeration system because the input in process 2-3 (Fig. 2) is usually wasted. The performance term in the refrigeration cycle is called the coefficient of performance, (COP), defined as

$$COP_{ref} = \frac{\text{Useful Refrigeration}}{\text{Net Work}} = \frac{\text{Refrigerating Effect}}{\text{Net work}}$$

The two terms which make up the coefficient of performance ( $COP_{ref}$ ) must be in the same unit, so that the  $COP_{ref}$  is dimensionless. From Fig. 2, refrigerating effect,

$$Q_e = Tds = T_1 (S_1 - S_4) \text{ and}$$

$$\text{Network } W = Q_c - Q_e = T_2 (S_2 - S_3) - T_1 (S_1 - S_4).$$

The Net Work is the area enclosed in rectangle 1-2-3-4 (Fig. 2). An expression for the coefficient of performance of the Carnot refrigeration cycle is:

$$COP_{ref,Carnot} = \frac{Q_e}{W} = \frac{Q_e}{Q_c - Q_e}$$

$$COP_{ref,Carnot} = \frac{T_1 (s_1 - s_4)}{T_2 (s_2 - s_3) - T_1 (s_1 - s_4)}$$

$$s_2 - s_3 = s_1 - s_4, \text{ since } s_2 = S_1 \text{ and } S_3 = S_4$$

Therefore,  $COP_{ref,Carnot} = \frac{T_1 (s_1 - s_4)}{(T_2 - T_1)(s_1 - s_4)}$

$$COP_{ref,Carnot} = \frac{T_1}{T_2 - T_1} = \frac{T_2}{T_2 - T_1} \quad (5.1)$$

Where,  $T_e$  = temperature in the evaporator

$T_c$  = temperature in the condenser

## HEAT PUMP

For heat pump  $COP_{hp} = \frac{\text{Heat Rejected from the Cycle}}{\text{Net Work}}$

$$COP_{hp} = \frac{Q_c}{Q_c - Q_e} = \frac{T_2 (s_2 - s_3)}{T_2 (s_2 - s_3) - T_1 (s_1 - s_4)}$$

$$COP_{hp} = \frac{T_2}{T_2 - T_1} \quad \text{or} \quad \frac{T_c}{T_c - T_e} \quad (5.2)$$

The coefficient of performance of the Carnot cycles is entirely a function of the temperature limits and vary from zero to infinity. The thermodynamic analysis per unit mass of reversed Carnot cycle with vapour as a refrigerant is given below:

Refrigerating effect,  $Q_e = h_1 - h_4$

Heat rejected,  $Q_c = h_2 - h_3$

Compressor Work,  $W_{cp} = h_2 - h_1$

Expander Work,  $W_{ex} = h_3 - h_4$

Net work,  $W = W_{cp} - W_{ex} = (h_2 - h_1) - (h_3 - h_4)$

Or  $W = Q_c - Q_e = (h_2 - h_3) - (h_1 - h_4)$

$$COP_{ref,Carnot} = \frac{Q_e}{Q_c - Q_e} = \frac{h_1 - h_4}{(h_2 - h_3) - (h_1 - h_4)} \quad (5.3)$$

## 5.4 TONS OF REFRIGERATION

It was common practice to measure amounts of refrigeration in tons of refrigeration. One ton of refrigeration (abbreviation: TR) is the amount of cooling produced by one U.S. ton of ice in melting over a period of 24 hours. Since an American ton is 907.2 kg and the latent of fusion of water amounts to 334.9 kJ/kg, therefore,

$$1 \text{ TR} = \frac{907.2 \times 334.9}{24 \times 3600} = 3.5165 \text{ kW}$$

If cooling required is  $x$  kW of refrigeration, the rate of refrigerant circulation necessary is as

$$\dot{m} = \frac{\text{Useful Refrigeration in kW}}{\text{Refrigerating Effect in kJ/kg}} = \frac{Q_c}{Q_e}$$

$$\dot{m} = \frac{W}{h_1 - h_4} \quad (5.4)$$

where  $\dot{m}$  is the mass flow rate of the refrigerant

## 5.5 ACTUAL REFRIGERATION SYSTEMS

Although the Carnot cycle is theoretically the most efficient cycle between given temperatures  $T_c$  and  $T_e$ , it sometimes has limitations for practical use. In actual cycle, the COP will be less than its Carnot value. For the purpose of comparison between the actual and Carnot value, the index of performance for cooling or refrigerating efficiency ( $\eta_{\text{ref}}$ ) is often used.

$$\text{Refrigerating Efficiency, } \eta_{\text{ref}} = \frac{\text{COP}_{\text{ref(actual)}}}{\text{COP}_{\text{ref(carnot)}}} \quad (5.5)$$

The reverse Carnot cycle with vapour as a refrigerant can be used as a practical cycle with minor modifications. The isothermal processes of heat rejection and heat absorption, accompanying condensation and evaporation respectively, are nearly perfect process and easily achievable in practice. But the isentropic compression and expansion process have limitations which are suitably modified as follows:

- (a) The reverse Carnot cycle is simplified by replacing the expansion cylinder with a simple throttle valve. Throttling process occurs such that the initial enthalpy equals the final enthalpy. The process is highly irreversible so that the whole cycle becomes irreversible. The process is represented by line 3 - 4 on Fig. 3.
- (b) The compression process in the Carnot cycle involves the compression of wet-refrigerant vapour. The wet compression is not found suitable in the practical refrigeration; therefore it is desirable to have refrigerant vapour initially dry at the start of compression as shown in Fig. 5.3. Such compression is known as dry compression by line 1-2 in Fig. 5.3. The state of the vapour at the end of compression will be at pressure  $p_c$  which is the saturation pressure of the refrigerant corresponding to the condensing temperature  $T_c$ . The wet compression is not found suitable due to the following reason:
- (i) Liquid refrigerant may be trapped in the head of the cylinder and may damage the compression valve and the cylinder itself.
  - (ii) Liquid- refrigerant droplets may wash away the lubricating oil from the walls of the compressor cylinder, thus increasing wear.

## SIMPLE SATURATION REFRIGERATION

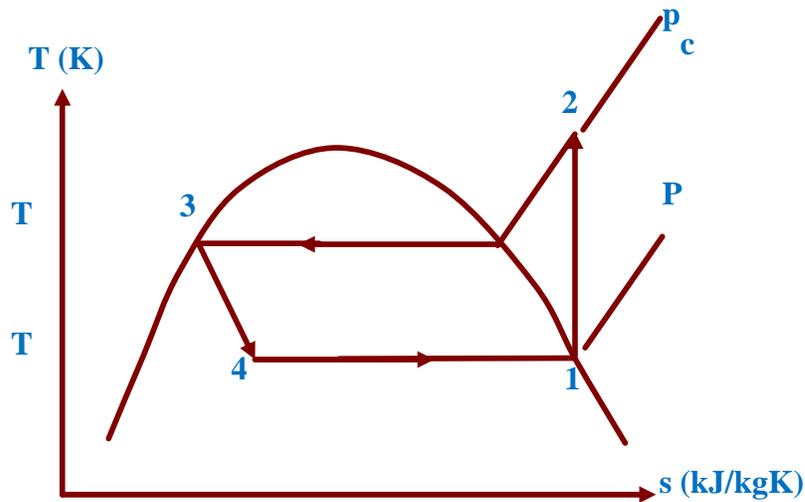


Fig. 5.3 T-s diagram of a simple saturation refrigeration cycle

### Expansion process 3-4:

Liquid refrigerant at condensing pressure  $p_c$  flows through expansion valve where its pressure falls to  $p_c$ , which is the pressure in the evaporator and at compressor suction. At the expansion valve part of the liquid refrigerant flashes into vapour (about 10-20%) vapour produced in this way is known as 'flash gases'. Point 4 is the state of refrigerant liquid and vapour mixture entering the evaporator. The process at the expansion valve is isenthalpic (constant enthalpy;  $h_3 = h_4$ )

### Evaporation process 4—1:

At the evaporator, the vaporization of the remaining liquid occurs at constant temperature  $T_e$ .

$$\text{Refrigerating effect} = h_1 - h_4 \quad (5.6)$$

**Compression process 1-2:**

Compression of refrigerant vapour from  $P_e$  to  $P_c$  occurs during process 1-2. As vapour is compressed the temperature rises and, finally, leaves the compressor as superheated gas at point 2. Work of compression  $W_c$  is given as:

$$\begin{aligned}W_c &= h_2 - h_1 \text{ (kJ/kg)} \\W'_c &= m'(h_2 - h_1) \quad (5.7)\end{aligned}$$

Where  $m'$  = mass of refrigerant circulated in kg/s.

**Condensation process 2-3:**

The refrigerant vapour loses its super heat and latent heat of evaporation. At constant pressure  $p_c$  all vapour will turn to liquid. The total amount of heat rejected to the condenser is given as

$$Q_c = h_2 - h_3 \text{ (kJ/kg)} \quad \text{or} \quad Q'_c = m'(h_2 - h_3) \text{ kW} \quad (5.8)$$

This cycle as described is called the simple saturation cycle. In practice:

- (i) The liquid entering the expansion valve is usually several degrees cooler than condensing temperature.
- (ii) The gas entering the compressor, on the other hand, is several degrees warmer than the temperature of evaporation.
- (iii) There are pressure drops in the suction, discharge and liquid pipelines.
- (iv) The compression process is not truly isentropic.
- (v) The actual power required to drive the compressor is somewhat greater than  $W_c$  on account of frictional losses.

All these factors have to be taken into a more exact quantitative treatment of the subject.

## 5.6 SYSTEM CAPACITY

The capacity of a system is usually expressed as “the rate at which heat is removed from the refrigerated space” usually expressed in kW. For a mechanical refrigerating system the capacity depends on two factors:

- (i) the mass of refrigerant circulated per unit of time
- (ii) the refrigerating effect per unit mass of refrigerant circulated.

$$Q'_e = m'Q_e \quad (5.9)$$

where,  $Q'_e =$  refrigerating capacity in kW or kJ/s

$Q_e =$  refrigerating effect in kJ/kg

$m' =$  mass flow rate in kg/s

$$V = m'v \quad (5.10)$$

where  $V =$  total volume of vapour generated in the evaporation in  $m^3/s$

$v =$  specific volume of the vapour at the vaporizing temperature in  $m^3/kg$